Simultaneous Linear Equations and Factorisation

Essential information

Simultaneous linear equations

These are of the form

$$ax + by = c$$
$$dx + ey = f$$

when a, b, c, d, e and f are constants.

Expanding brackets

Expressions such as $a(x + b)$ are expanded out.

For example,

$$2(x + 3) = 2x + 6$$
$$x(4 - x) = 4x - x^2$$
$$2x(1 + 3x) = 2x + 6x^2$$
$$a(x + b) = ax + ab$$

- $(a + b)(a - b) = a^2 - b^2$
- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- If $x^2 + ax + b = (x + p)(x + q)$, then
  $a = p + q$ and $b = pq$

Linear factorisation

Single factors are taken out of an expression.

For example,

$$3x + 9 = 3(x + 3)$$
$$4x^2 + x = x(4x + 1)$$
$$5x^2 + 10x = 5x(x + 2)$$

Quadratic factorisation

Quadratic expressions are factorised into linear factors.

For example,

$$x^2 + 5x + 6 = (x + 3)(x + 2)$$
$$x^2 - 4 = (x + 2)(x - 2)$$
$$(2x - 1)(x + 2) = 2x^2 + 3x - 2$$

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Also note that, in general, simultaneous linear equations normally have unique solutions, but not when one equation is the multiple of another. For example, when

$$\begin{aligned} x \ + \ y \ &= 1 \\ 2x \ + \ 2y \ &= 2 \end{aligned}$$

there is no unique solution. Also when there are two inconsistent equations, for example,

$$\begin{aligned} x + y &= 1 \\ x + y &= 3 \end{aligned}$$

there are NO solutions.