Sequences, Series and Sigma Notation Key points

Key points and concepts

- The general form of a geometric sequence of n terms is

  $$a,\ ar,\ ar^2,\ \dots,\ ar^{n-1}$$

  where a is a constant and r, the ratio of consecutive terms, is called the *common ratio.*

- The sum of n terms of a geometrical series is given by

  $$S_n = \frac{a\left(r^n - 1\right)}{\left(r - 1\right)} \qquad \left(r \neq 1\right)$$

- If $|r| < 1$, the sum to infinity of a geometric series is given by

  $$S_n \to \frac{a}{\left(1 - r\right)} \quad \text{as} \quad n \to \infty$$

- The general arithmetic sequence is of the form

  $$a,\ a + d,\ a + 2d,\ a + 3d,\ \dots$$

  where a and d are constants.

- The sum of n terms of an arithmetic sequence is given by

  $$S_n = \frac{n}{2}\left(2a + \left(n - 1\right)d\right)$$

  If l is the last term, then $l = a + \left(n - 1\right)d$ and

  $$S_n = \frac{n}{2}\left(a + l\right)$$

- Sigma notation can be used to write a sum of series.

  For example, $2 + 4 + 8 + \dots + 2^{12} = \sum_{r=1}^{12} 2^r$

- Important results are

  $$\sum_{r=1}^{n} r = \frac{1}{2}n\left(n + 1\right)$$

  $$\sum_{r=1}^{n} r^2 = \frac{1}{6}n\left(n + 1\right)\left(2n + 1\right)$$

  $$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2\left(n + 1\right)^2$$