RELECTIONS, ROTATIONS AND ENLARGEMENTS

Text

Contents

Section

1 Reflections

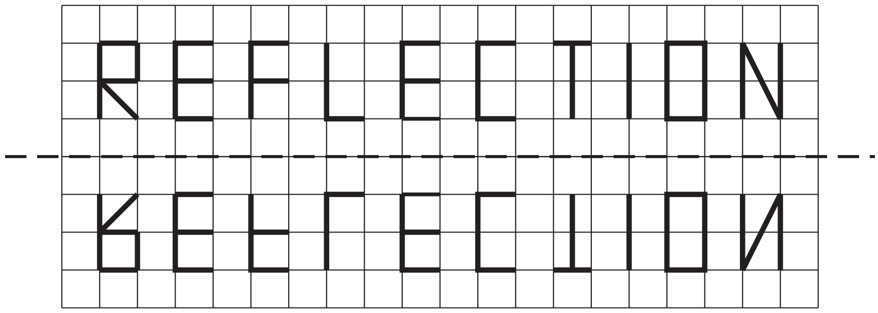
2 Rotations

3 Enlargements

Reflections, Rotations and Enlargements

1 Reflections

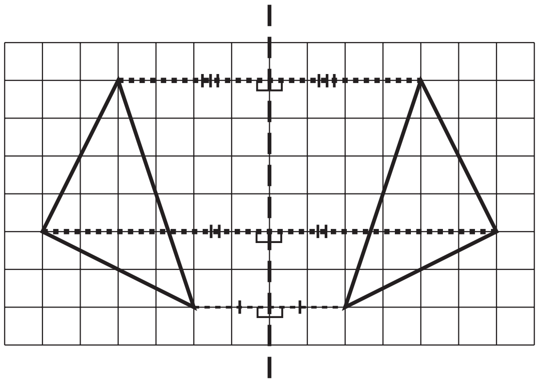
Reflections are obtained when you draw the image that would be obtained in a mirror.



Mirror Line

Every point on a reflected image is always the same distance from the *mirror line* as the original. This is shown below.

Mirror Line



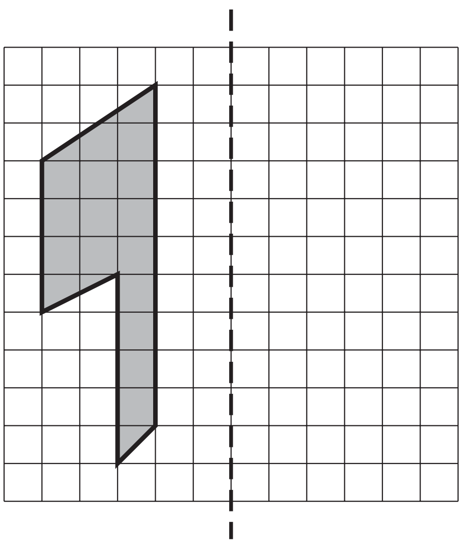
Note

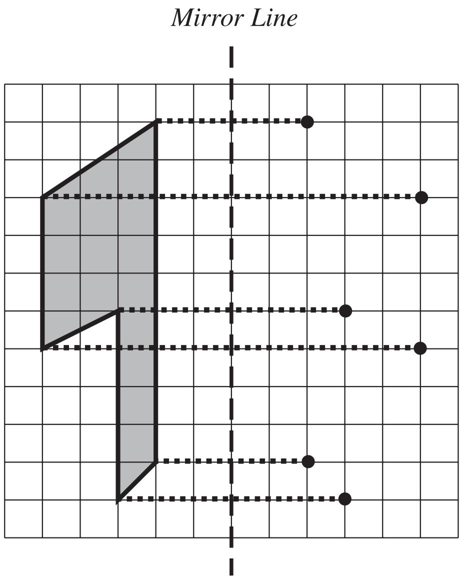
Distances are always measured at right angles to the *mirror line*.

Worked Example 1

Draw the reflection of the shape in the mirror line shown.

Mirror Line

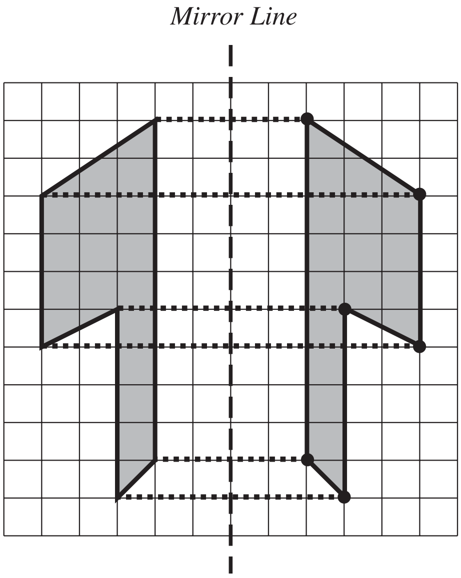




Solution

The lines added to the diagram show how to find the position of each point after it has been reflected.

Remember that the image of each point is the same distance from the mirror line as the original.

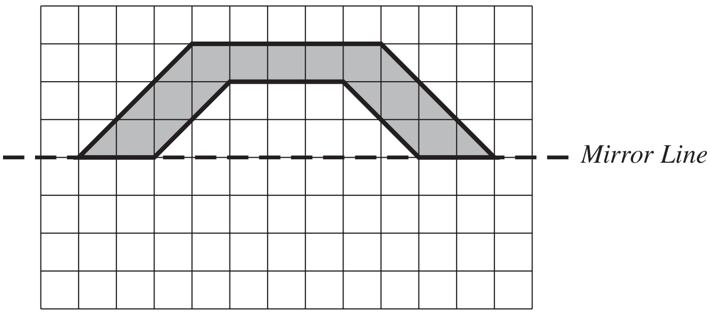


The points can then be joined to give the reflected image.

If the construction lines have been drawn in pencil they can be rubbed out.

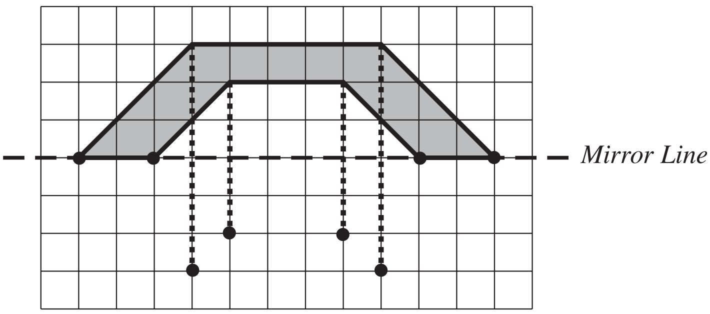
Worked Example 2

Reflect this shape in the mirror line shown in the diagram.

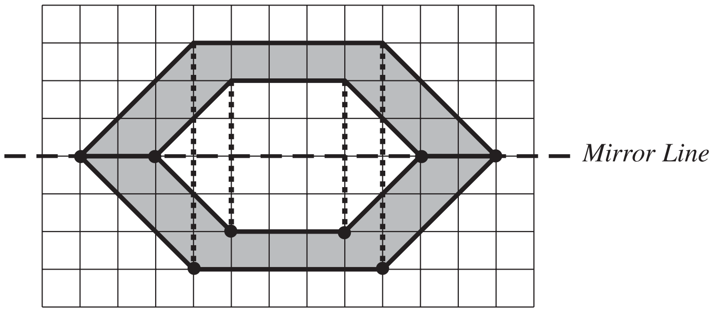


Solution

The lines are drawn at right angles to the mirror line. The points which form the image must be the same distance from the mirror lines as the original points. The points which were on the mirror line remain there.



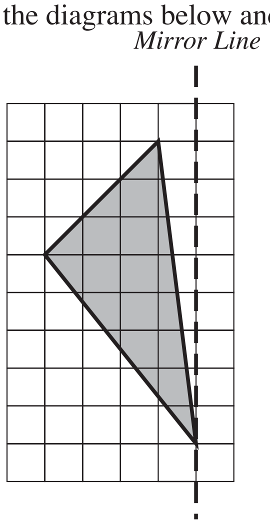
The points can then be joined to give the reflected image.



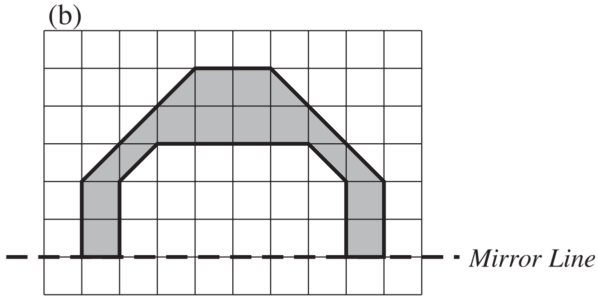
Exercises

1. Copy the diagrams below and draw the reflection of each object.

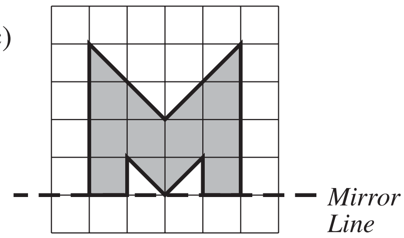
(a)



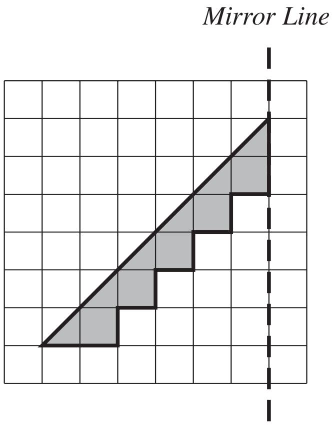
(b)



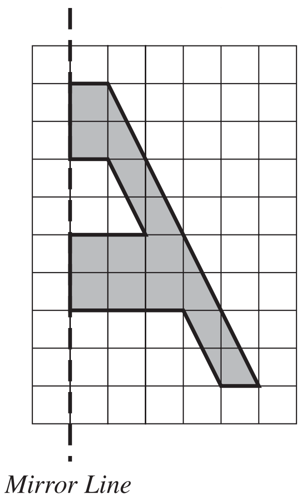
(c)



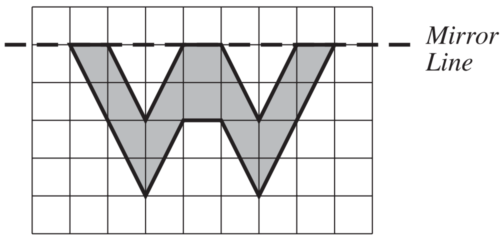
(d)



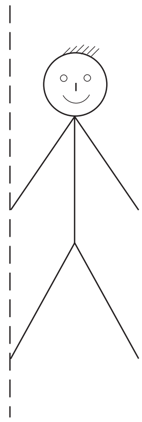
(e)

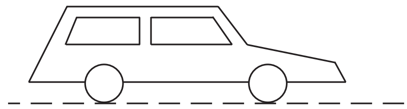


(f)



2. Copy each diagram and draw the reflection of each shape in the mirror line shown.

(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

3. (a) Draw a set of axes with x and y values from -5 to 5.

(b) Plot the points with coordinates

$$(1, 1),\ (1, 5),\ (4, 5),\ (4, 3),\ (2, 3),\ (2, 1).$$

Join the points in that order to form a shape.

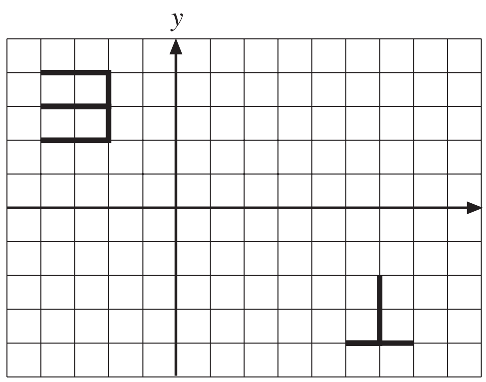
(c) Reflect the image in the y-axis. Write down the coordinates of the corners of this shape.

(d) Reflect the image obtained in (c) in the x-axis. List the coordinates of the corners.

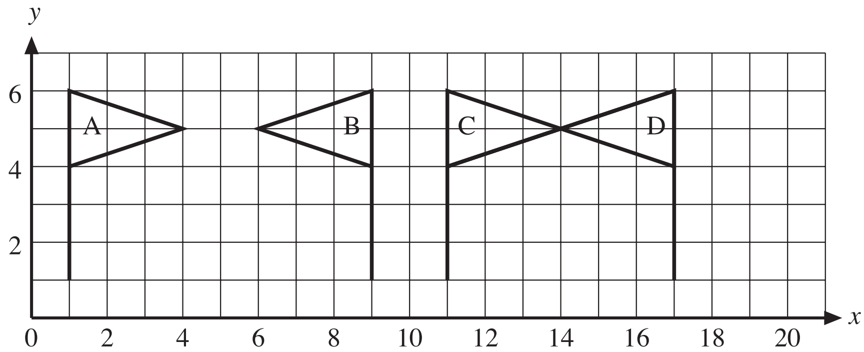
(e) Reflect the image obtained in (d) in the y-axis. Describe how this shape could have been obtained directly from the original shape.

4. A student reflected his two initials, the first in the y-axis and the second in the x-axis, to obtain the image opposite.

Copy the diagram and show the original position of the initials.



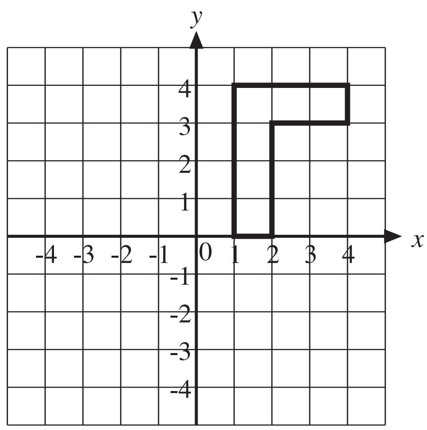
5. Copy the diagram below.



Draw in the mirror line for each reflection described.

(a) A → B (b) B → C (c) C → D (d) A → D

6. (a) Copy the axes and shape shown.



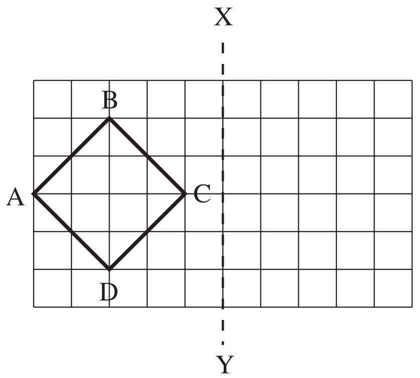
(b) (i) Draw the reflection of the shape in the *y*-axis.

(ii) Compare the coordinates of each shape.

(iii) Describe what happens to the coordinates of a point when it is reflected in the *y*-axis.

(c) Repeat (b) using the *x*-axis.

7.

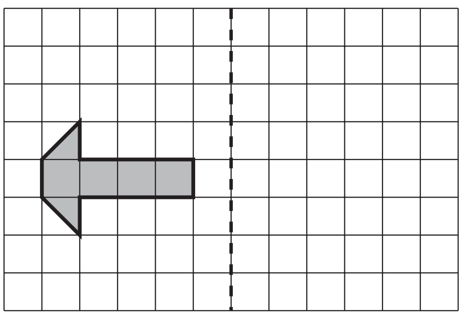


(a) Copy the diagram and draw the reflection of ABCD in the mirror line XY.

(b) ABCD has rotational symmetry. Mark with a cross its centre of rotation.

8. (a) Find the area of the shaded shape.

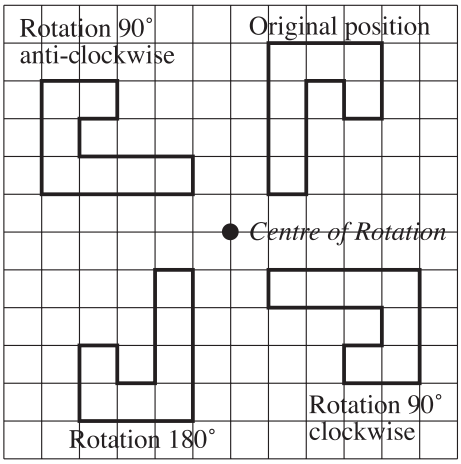
(b) Copy the diagram and draw the reflection of the shaded shape in the mirror line.



Mirror Line

2 Rotations

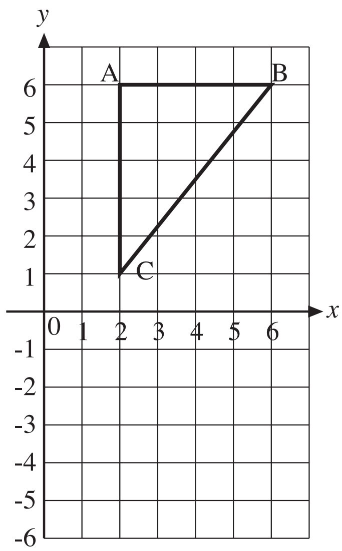
Rotations are obtained when a shape is rotated about a fixed point, called the *centre of rotation*, through a specified angle. The diagram shows a number of rotations.



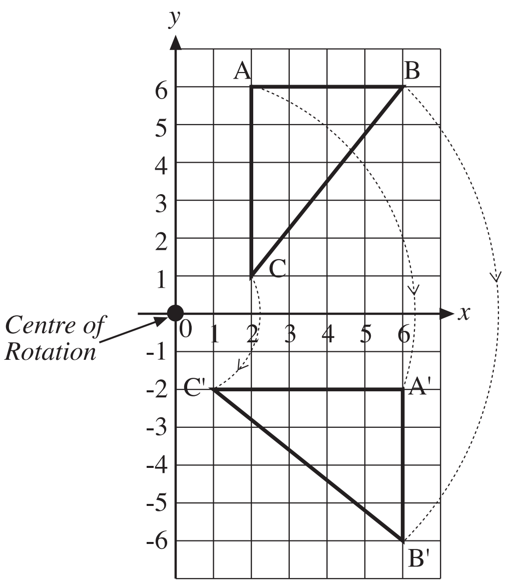
It is often helpful to use tracing paper to find the position of a shape after a rotation.

Worked Example 1

Rotate the triangle ABC shown in the diagram through $90°$ clockwise about the point with coordinates $(0, 0)$.



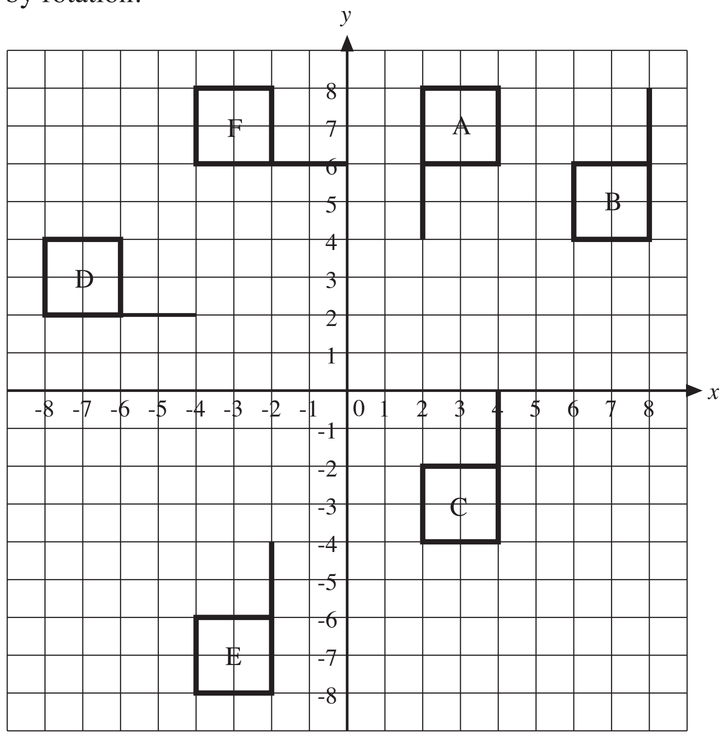
Solution



The diagram opposite shows how each vertex can be rotated through $90°$ to give the position of the new triangle.

Worked Example 2

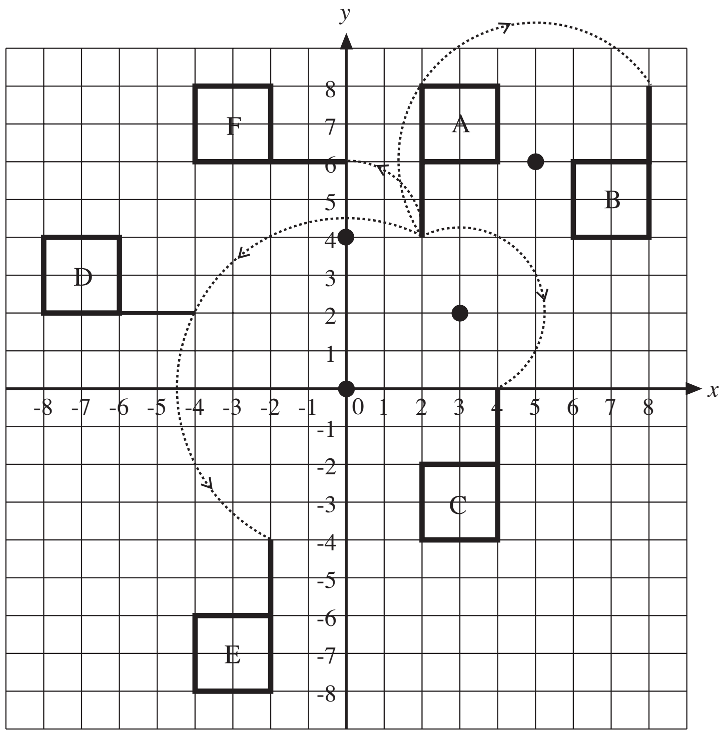
The diagram shows the position of a shape A and the shapes, B, C, D, E and F which are obtained from A by rotation.



Describe the rotation which moves A onto each other shape.

Solution

The diagram shows the centres of rotation and how one vertex of the shape A was rotated.



Each rotation is now described.

- A to B: Rotation of $180°$ about the point $(5, 6)$.
- A to C: Rotation of $180°$ about the point $(3, 2)$.
- A to D: Rotation of $90°$ anti-clockwise about the point $(0, 0)$.
- A to E: Rotation of $180°$ about the point $(0, 0)$.
- A to F: Rotation of $90°$ anti-clockwise about the point $(0, 4)$.

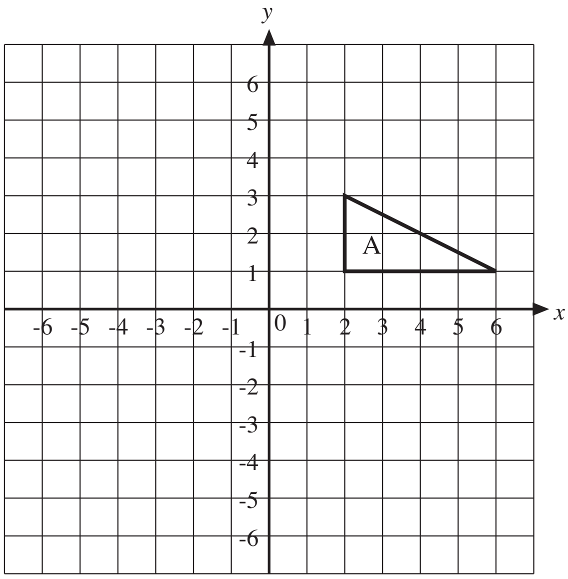
Exercises

1. Copy the axes and triangle shown opposite.

   (a) Rotate A through $90°$ clockwise around $(0, 0)$ to obtain B.

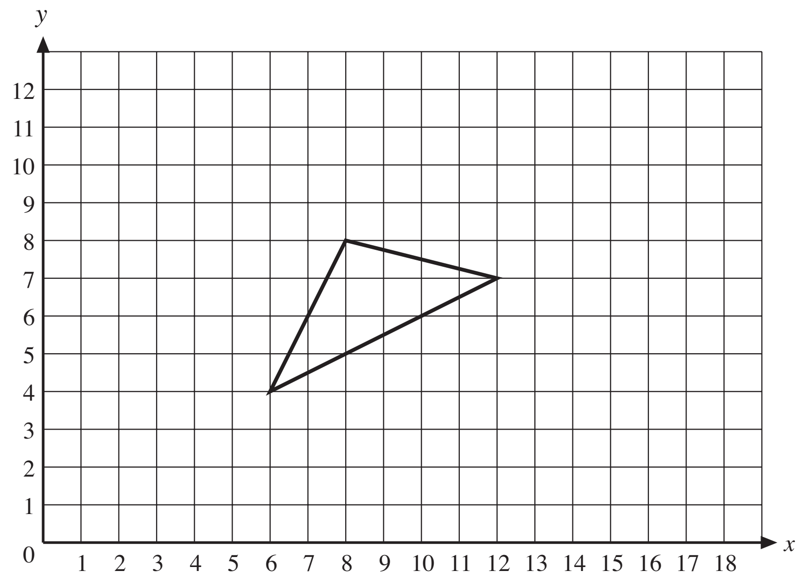
   (b) Rotate A through $90°$ anticlockwise around $(0, 0)$ to obtain C.

   (c) Rotate A through $180°$ around $(0, 0)$ to obtain D.



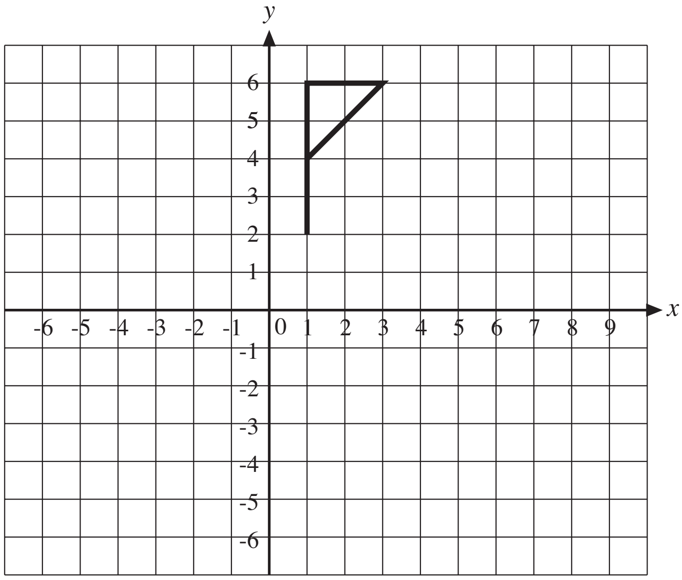
2. Repeat *Question 1* for the triangle with coordinates $(3, 1)$, $(6, 2)$ and $(0, 4)$.

3. Copy the axes and triangle shown below.



Rotate the triangle through $180°$ using each of its vertices as the centre of rotation.

4. Copy the axes and shape shown below.



(a) Rotate the original shape through $90°$ clockwise around the point $(1, 2)$.

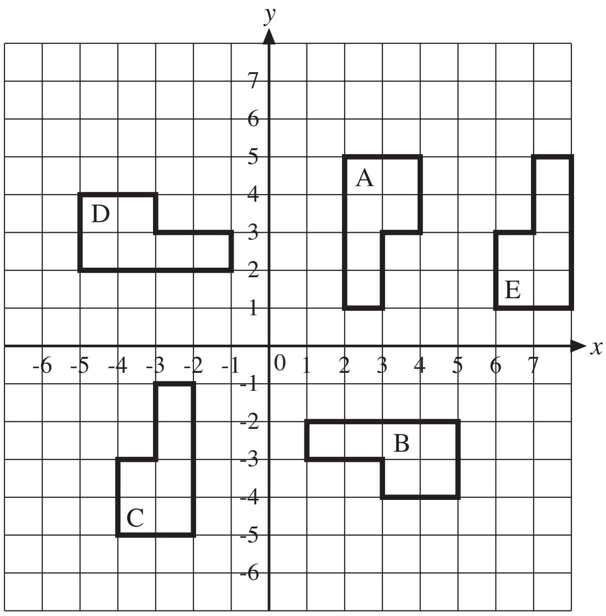
(b) Rotate the original shape through $180°$ around the point $(3, 4)$.

(c) Rotate the original shape through $90°$ clockwise around the point $(1, -2)$.

(d) Rotate the original shape through $90°$ anti-clockwise around the point $(0, 1)$.

5. Repeat *Question 4* for the triangle with vertices at $(2, 2)$, $(1, 3)$ and $(3, 5)$.

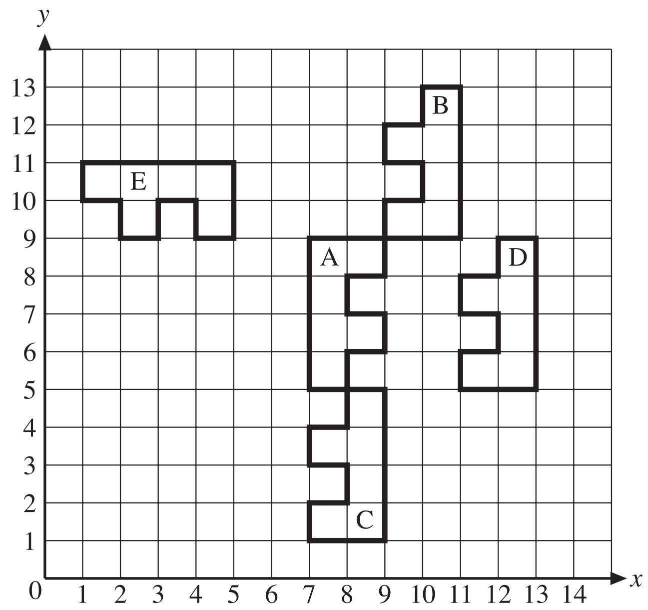
6. The diagram shows the position of a shape labelled A and other shapes which were obtained by rotating A.



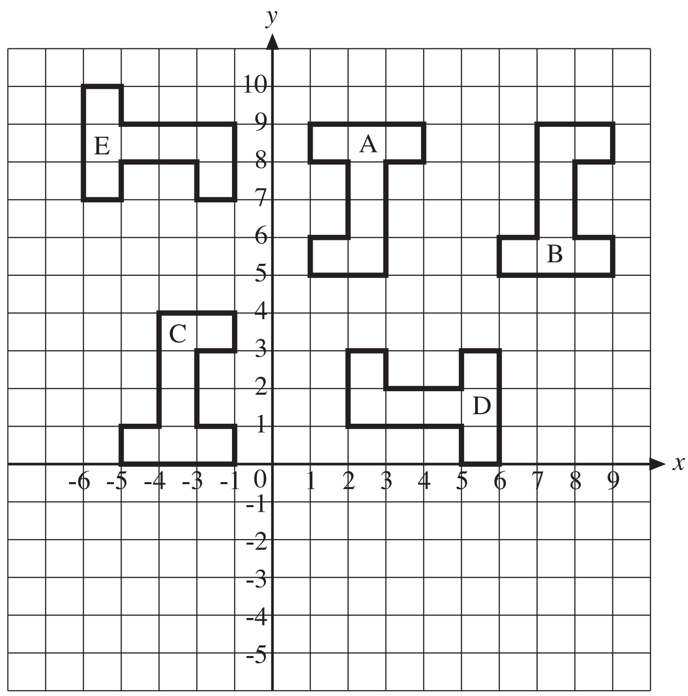
(a) Describe how each shape can be obtained from A by a rotation.

(b) Which shapes can be obtained by rotating the shape E?

7. The shape A has been rotated to give each of the other shapes shown. For each shape, find the centre of rotation.



8. (a) Describe how each shape shown below can be obtained from A by a rotation.



(b) Which shapes cannot be obtained from C by a rotation?

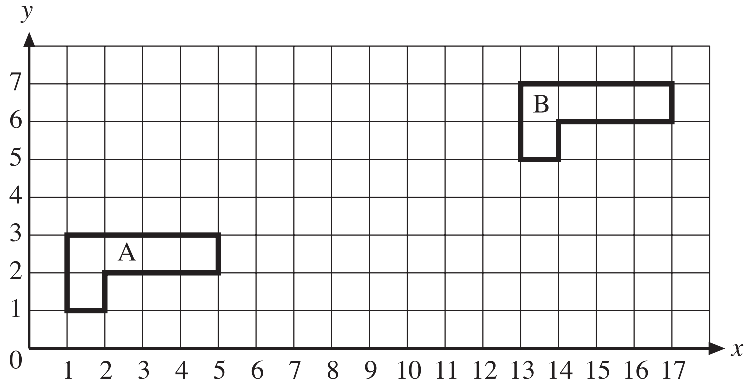
9. On a set of axes with x and y values from -2 to 12, draw the triangle with vertices at the points $(0, 0)$, $(4, 5)$ and $(1, 4)$.

   (a) Rotate the triangle through $90°$ clockwise about the point $(5, 6)$.

   (b) Rotate the second triangle through $90°$ clockwise about the point $(4, 3)$.

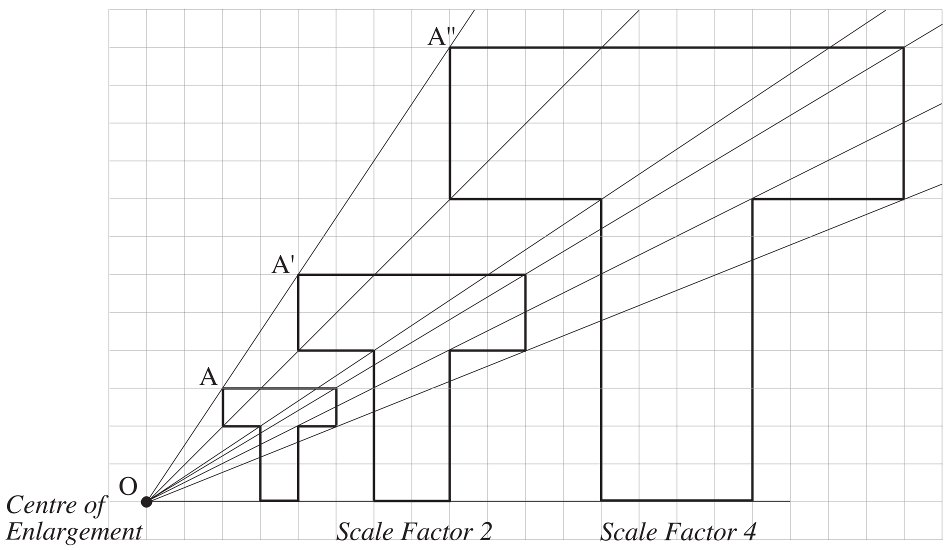
   (c) Describe how to obtain the third triangle from the original triangle by a single rotation.

10. The shape B can be obtained from A by two rotations. Describe these rotations.



3 Enlargements

An *enlargement* is a transformation which enlarges (or reduces) the size of an image. Each enlargement is described in terms of a *centre of enlargement* and a *scale factor*.



The example shows how the original, A, was enlarged with scale factors 2 and 4. A line from the *centre of enlargement* passes through the corresponding vertex of each image.

Note

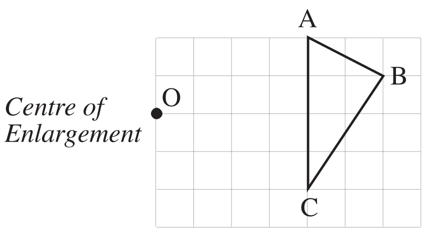
The distances, OA′ and OA″, are related to OA:

$$OA' = 2 \times OA$$
$$OA'' = 4 \times OA$$

The same is true of all the other distances between O and corresponding points on the images.

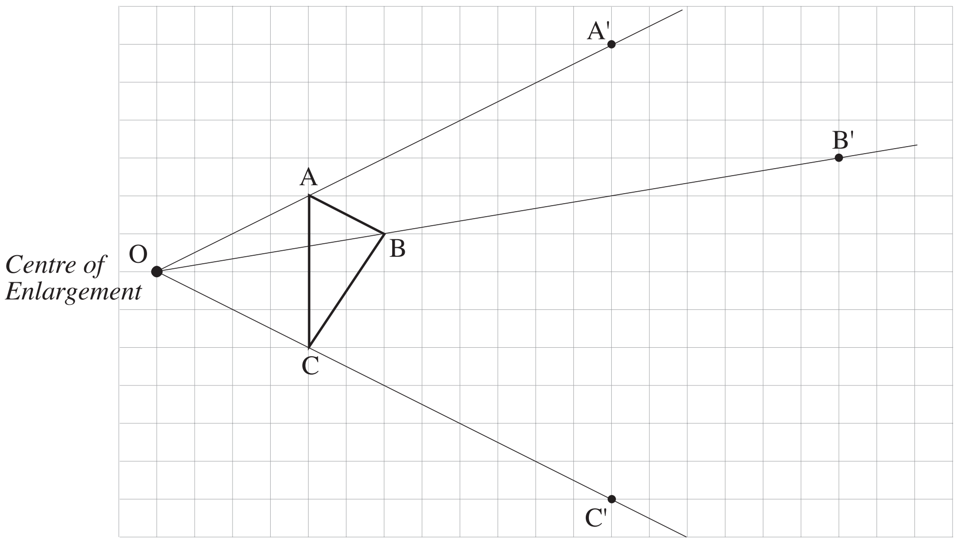
Worked Example 1

Enlarge the triangle shown using the centre of enlargement marked and scale factor 3.



Solution

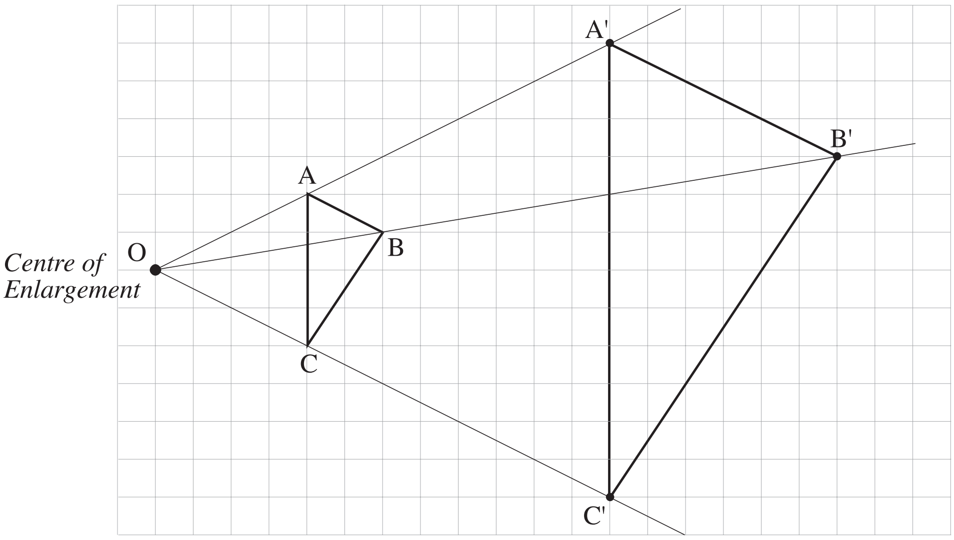
The first step is to draw lines from the centre of enlargement through each vertex of the triangle as shown below.



As the scale factor is 3, then

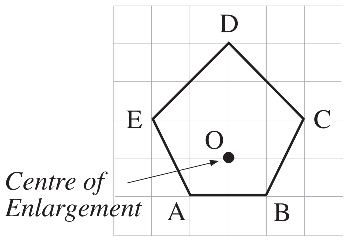
$$OA' = 3 \times OA$$
$$OB' = 3 \times OB$$
$$OC' = 3 \times OC$$

The points A', B' and C' have also been marked on the diagram. Once these points have been found they can be used to draw the enlarged triangle.



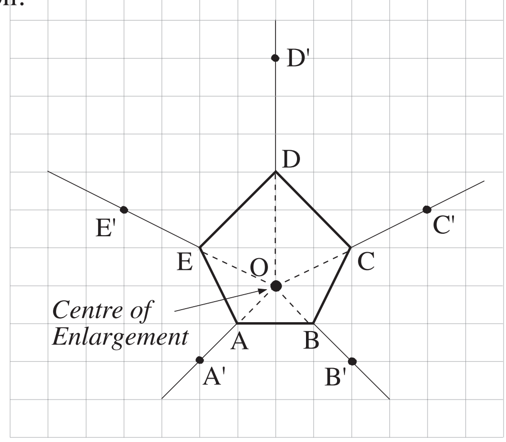
Worked Example 2

Enlarge the pentagon with scale factor 2 using the centre of enlargement marked on the diagram.



Solution

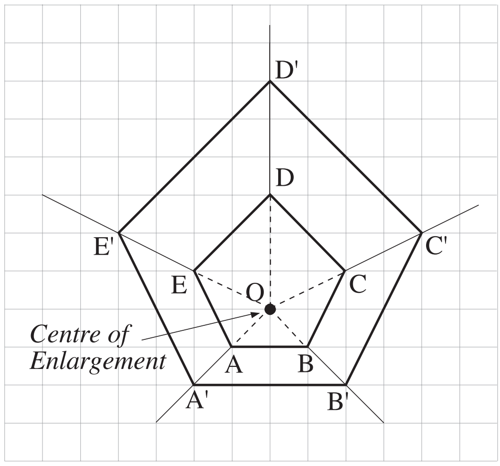
The first step is to draw lines from the centre of enlargement which pass through the five vertices of the pentagon.



As the scale factor is 2 the distances from the centre of enlargement to the vertices of the image will be

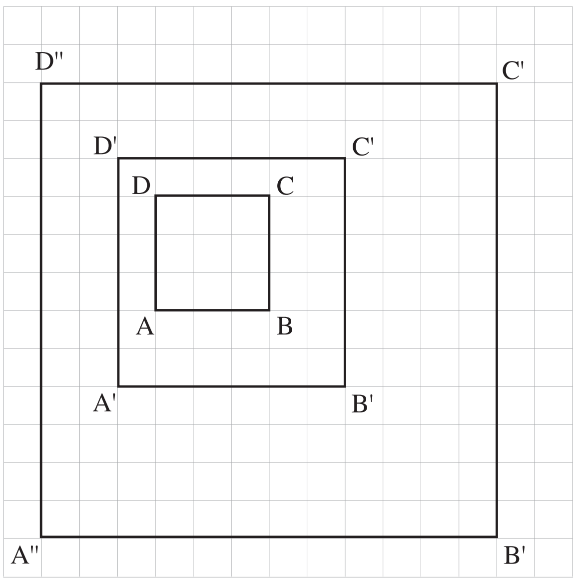
$$OA' = 2 \times OA$$
$$OB' = 2 \times OB$$
$$OC' = 2 \times OC$$
$$OD' = 2 \times OD$$
$$OE' = 2 \times OE\,.$$

These points can then be marked and joined to give the enlargement.



Worked Example 3

The diagram shows the square ABCD which has been enlarged to give the squares $A'B'C'D'$ and $A''B''C''D''$.



(a) Find the scale factor for each enlargement.

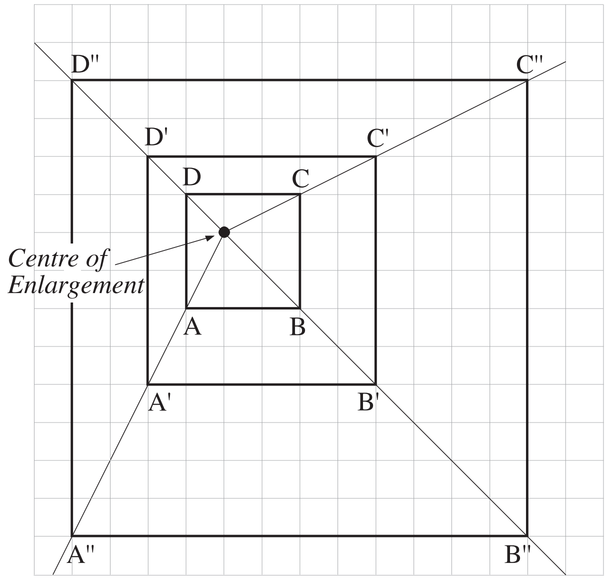
(b) Find the centre of enlargement.

Solution

(a) The sides of the square ABCD are each 1.5 cm. The sides of the square $A'B'C'D'$ are 3 cm. As these are twice as long as the original, the scale factor for this enlargement is 2.

The sides of the square $A''B''C''D''$ are 6 cm, which is 4 times longer than the original square. So the scale factor for this enlargement is 4.

(b) To find the centre of enlargement draw lines through A, A' and A'', then repeat for B, B' and B'', C, C' and C'' and D, D' and D''.



These lines cross at the centre of enlargement as shown in the diagram.

Note

When the scale factor of an enlargement is a fraction, the size of the enlargement is reduced. The image of the original is then between the *centre of enlargement* and the original.

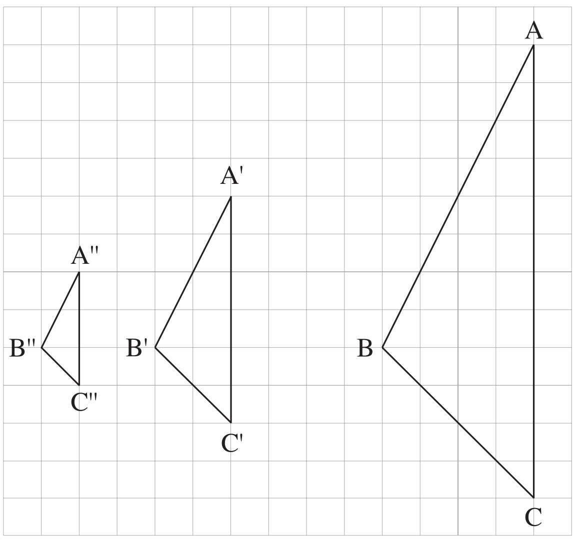
Worked Example 4

The diagram shows three triangles.

ABC was enlarged with different scale factors to give $A'B'C'$ and $A''B''C''$.

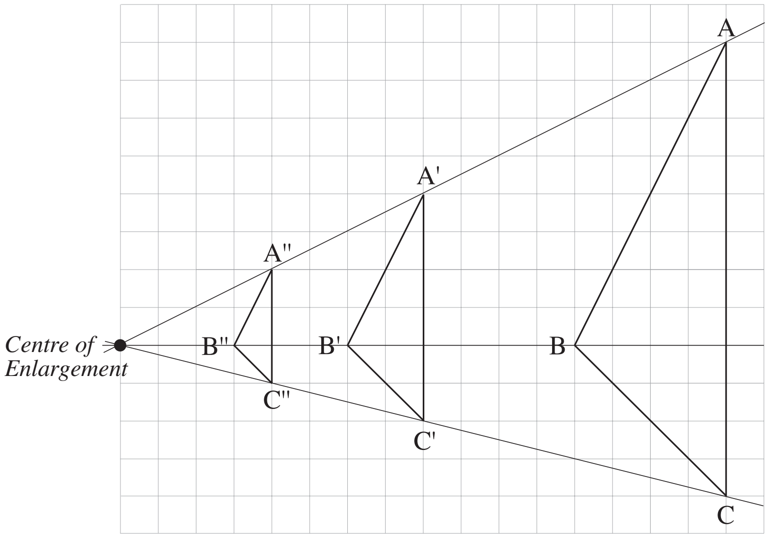
(a) Find the centre of enlargement.

(b) Find the scale factor for each enlargement.



Solution

(a) To find the centre of enlargement, lines should be drawn through the corresponding points on each figure.



(b) To find the scale factors, compare the lengths of sides in the different triangles. First consider triangles ABC and $A'B'C'$:

$$AC = 6 \text{ cm} \quad \text{and} \quad A'C' = 3 \text{ cm},$$

so $$A'C' = \frac{1}{2} \times AC$$

which means that the scale factor is $\frac{1}{2}$.

For triangles ABC and $A''B''C''$,

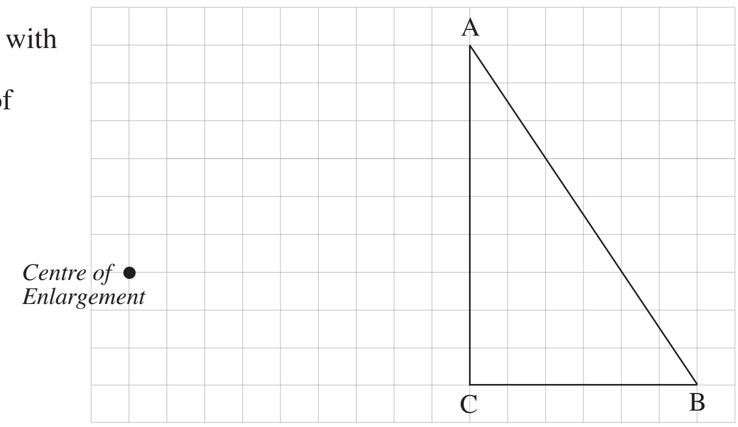
$$AC = 6 \text{ cm} \quad \text{and} \quad A''C'' = 1.5 \text{ cm},$$

so $$A''C'' = \frac{1}{4} \times AC$$

which means that the scale factor is $\frac{1}{4}$.

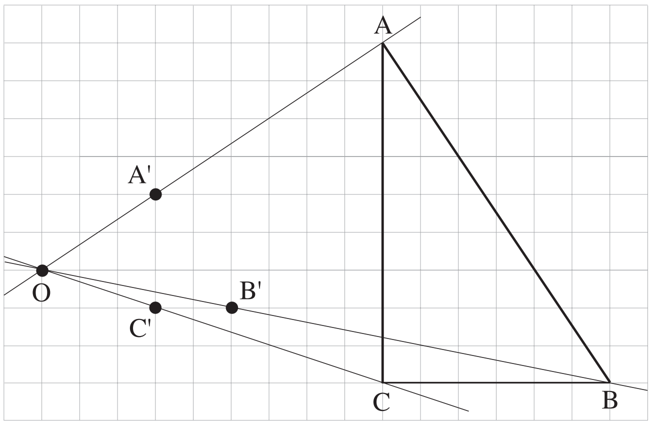
Worked Example 5

Enlarge the triangle shown with scale factor $\frac{1}{3}$ and centre of enlargement as shown.

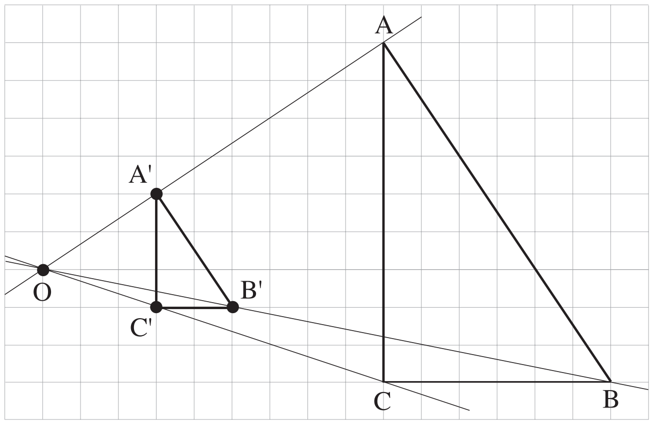


Solution

The first stage is to draw lines from each corner of the triangle through the centre of enlargement.



These points can then be joined to give the image.



Then the corners of the image should be fixed so that

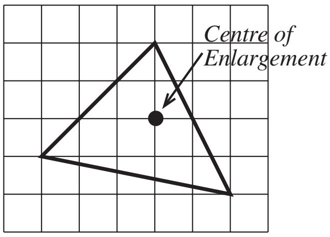
$$OA' = \frac{1}{3} \times OA$$

$$OB' = \frac{1}{3} \times OB$$

$$OC' = \frac{1}{3} \times OC$$

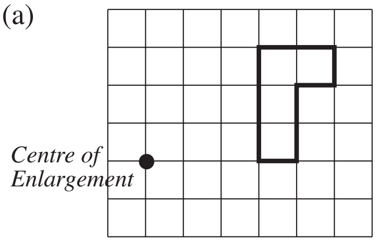
Exercises

1. Enlarge the triangle shown with scale factor 3 and the centre of enlargement shown.

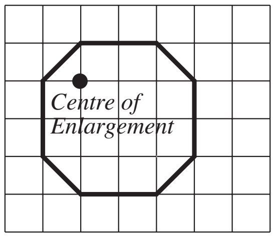


2. Copy the diagrams below on to squared paper. Enlarge each shape with scale factor 2 using the point marked as the centre of enlargement.

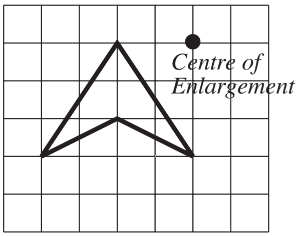
(a)



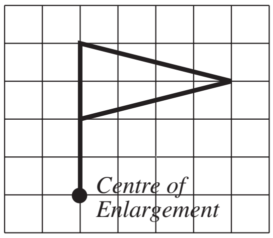
(b)



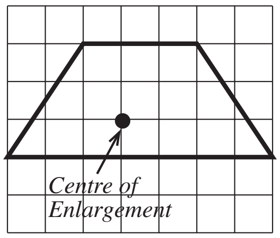
(c)



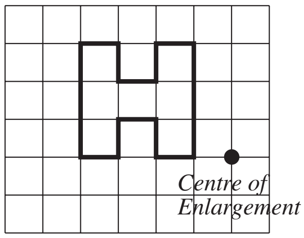
(d)



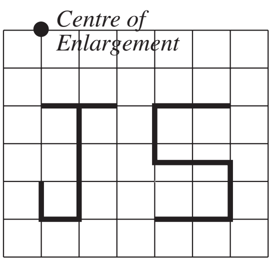
(e)



(f)



3. (a) A boy writes his initials as shown below.

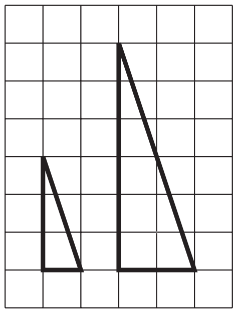


Use the marked centre of enlargement to enlarge his initials with scale factor

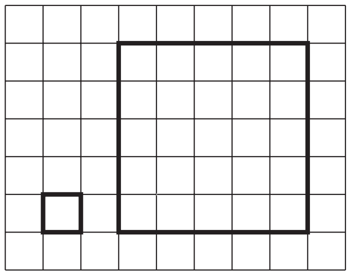
(b) Repeat part (a) using your own initials.

4. In each diagram below, the smaller shape has been enlarged to obtain the larger shape. For each example state the scale factor.

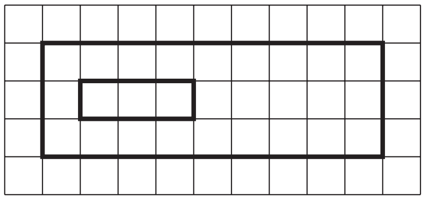
(a)



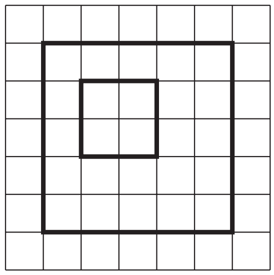
(b)



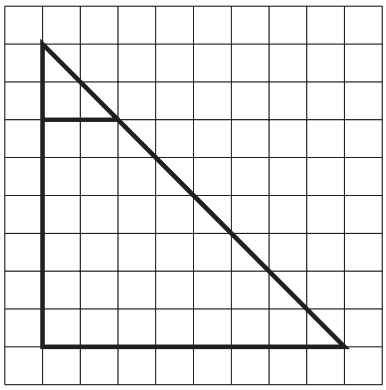
(c)



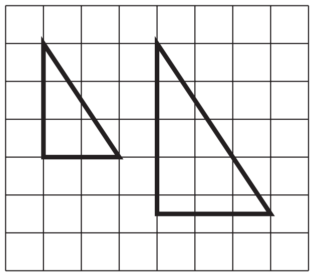
(d)



(e)

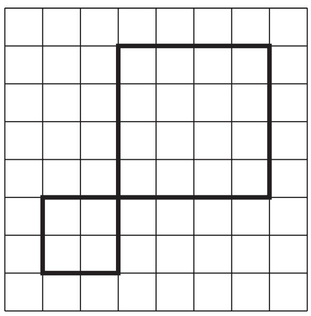


(f)

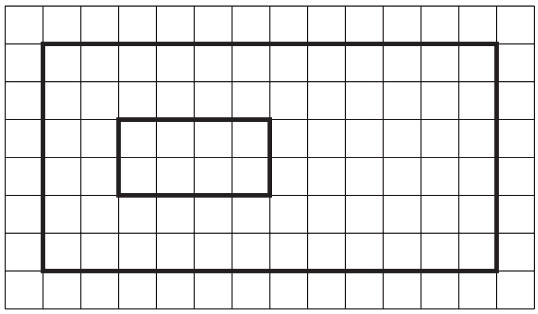


5. Copy each diagram on to squared paper. Then find the centre of enlargement and the scale factor when the smaller shape is enlarged to give the bigger shape.

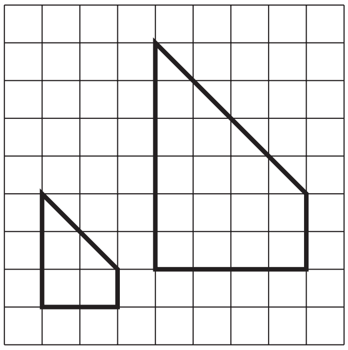
(a)



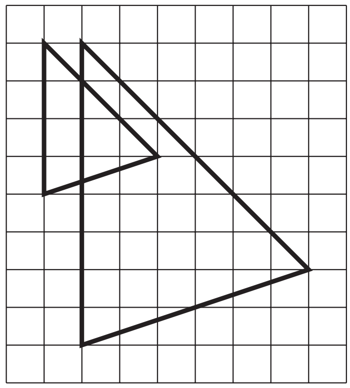
(b)



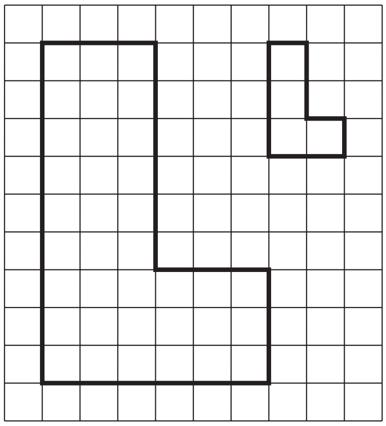
(c)



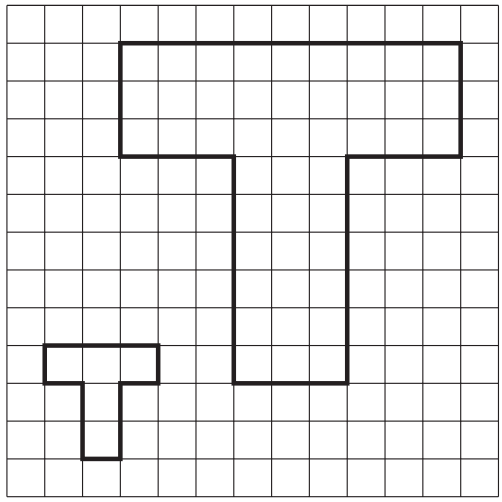
(d)



(e)

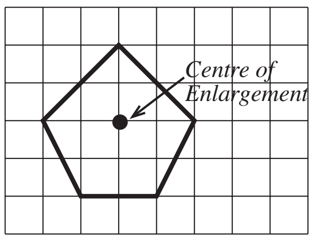


(f)

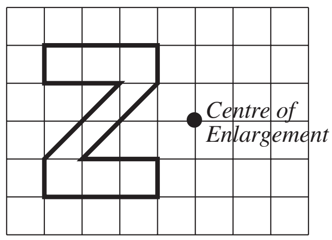


6. Copy each diagram below and enlarge it with scale factor 3.

(a)



(b)



7. (a) Draw a set of axes with *x* and *y* values from 0 to 15.

(b) Plot the points $(2, 2)$, $(6, 2)$, $(6, 6)$ and $(2, 6)$, then join them to form a square.

(c) Enlarge the square with scale factor 3, using the point with coordinates $(3, 3)$ as the centre of enlargement.

8. (a) Draw a circle with centre at $(4, 4)$ and radius 2.

(b) Enlarge the circle with scale factor 2 and centre of enlargement $(4, 4)$.

(c) Enlarge the circle with scale factor 3 and centre $(5, 5)$.

9. A triangle with vertices at the points with coordinates (2, 1) (7, 1) and (7, 6) is enlarged to give the triangle with coordinates at the points (6, 3), (21, 3) and (21, 18).

(a) Draw both triangles.

(b) What is the scale factor of the enlargement?

(c) What are the coordinates of the centre of enlargement?

10. (a) On a set of axes draw a triangle with vertices at (2, 0), (4, 0) and (3, 3).

(b) Enlarge the triangle with scale factor 2 using the point (0, 0) as the centre of enlargement.

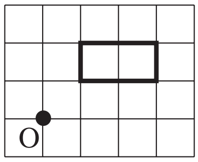
(c) Write down the coordinates of both triangles. How do they compare?

(d) What would you expect to be the coordinates of your triangle if it were to be enlarged with scale factor 3 using (0, 0) as the centre of enlargement?

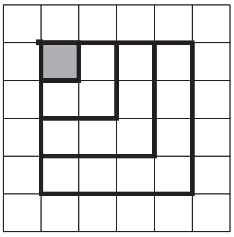
Check your answer by drawing the triangle.

(e) Enlarge your original triangle with a different centre. Is there a simple relationship between the coordinates of the original and the enlargement, when the centre of enlargement is used?

11. Using the point O as the centre of enlargement, enlarge the rectangle with scale factor 3.



12.



The shaded square has sides of length 1 cm.
It is enlarged a number of times as shown.

(a) Copy and complete the table below.

Length of Side of Square	1 cm	2 cm	3 cm	4 cm
Perimeter of Square	4 cm	8 cm	12 cm	
Area of Square	1 cm^2	4 cm^2		16 cm^2

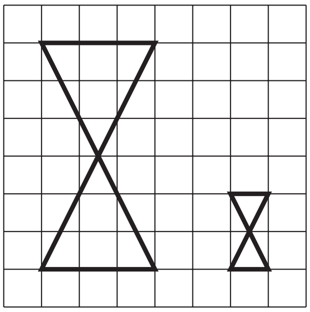
The shaded square continues to be enlarged.

(b) Copy and complete the following table.

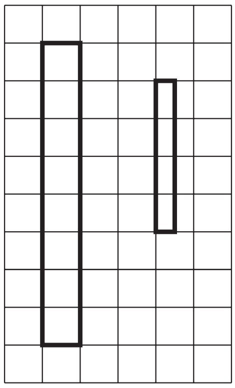
Length of Side of Square	
Perimeter of Square	
Area of Square	64 cm

13. For each pair of objects, state the scale factor of an enlargement which produces the smaller image from the larger one.

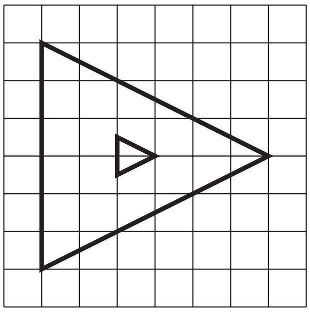
(a)



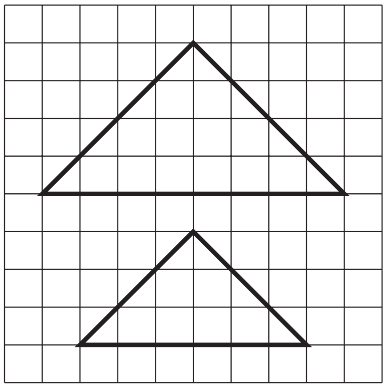
(b)



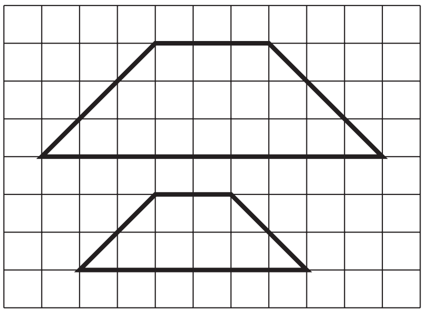
(c)



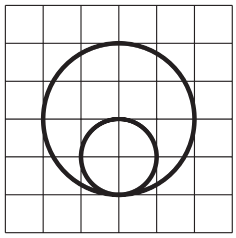
(d)



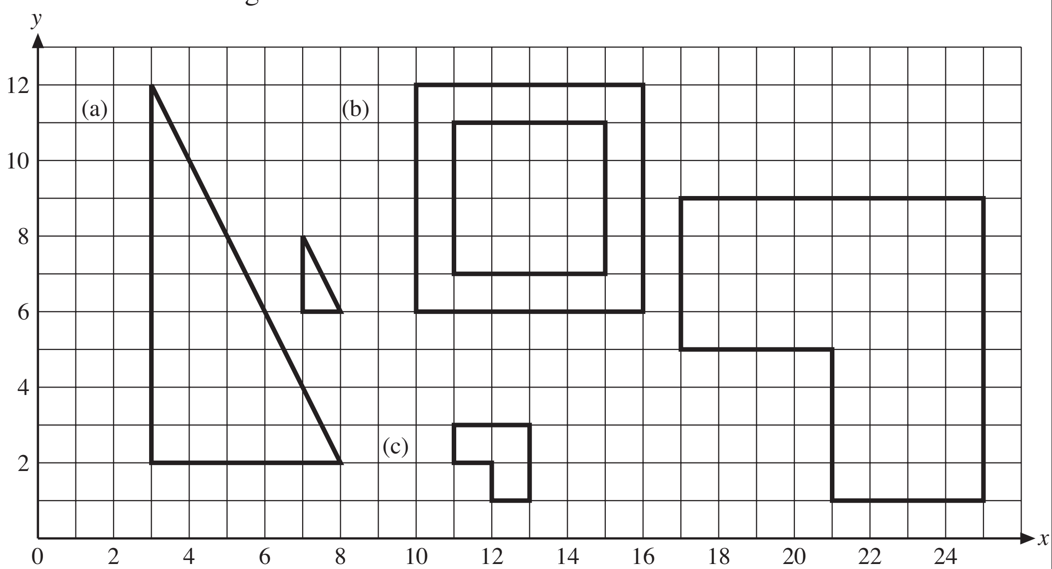
(e)



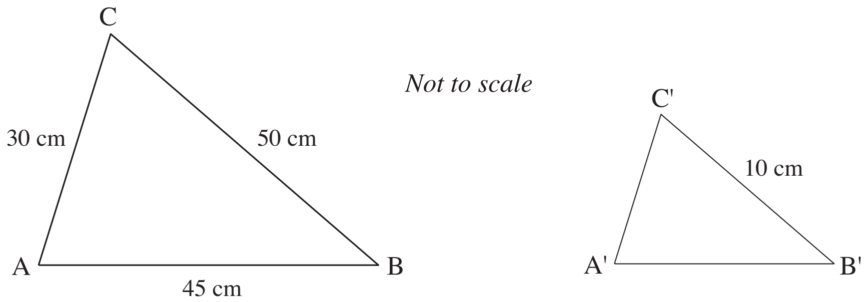
(f)



14. In each example below, the smaller shape has been obtained from the larger shape by an enlargement. For each example, state the scale factor and the coordinates of the centre of enlargement.



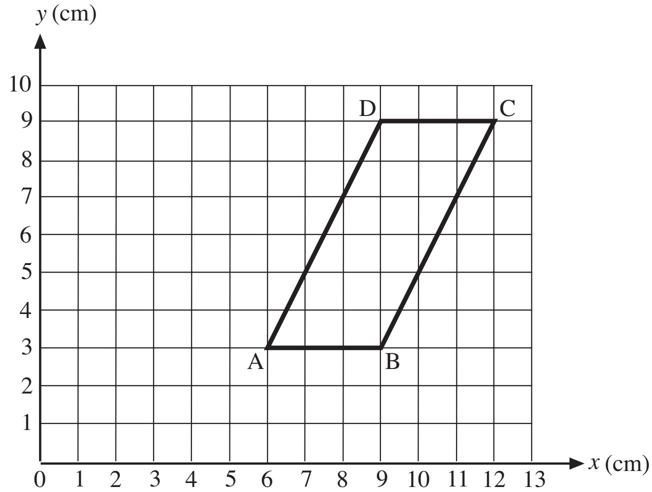
15. The larger triangle shown below is reduced in size by using a photocopier to give the smaller triangle.



(a) What is the scale factor of the enlargement which took place?

(b) What are the lengths of $A'C'$ and $B'C'$?

16. The parallelogram ABCD has vertices $(6, 3)$, $(9, 3)$, $(12, 9)$ and $(9, 9)$ respectively.



An enlargement scale factor $\frac{1}{3}$ and centre $(0, 0)$ transforms parallelogram ABCD onto parallelogram $A'B'C'D'$.

(a) (i) Draw the parallelogram $A'B'C'D'$.

(ii) Calculate the area of parallelogram $A'B'C'D'$.

(b) The side AB has length 3 cm. The original shape ABCD is now enlarged with a scale factor of $\frac{2}{5}$ to give $A''B''C''D''$.

Calculate the length of the side $A''B''$.

Investigation

Points X and Y lie on a straight line AB. Given that $AX : XB = 1 : 2$ and $AY : YX = 2 : 3$, write down the ratio $AY : XB$.