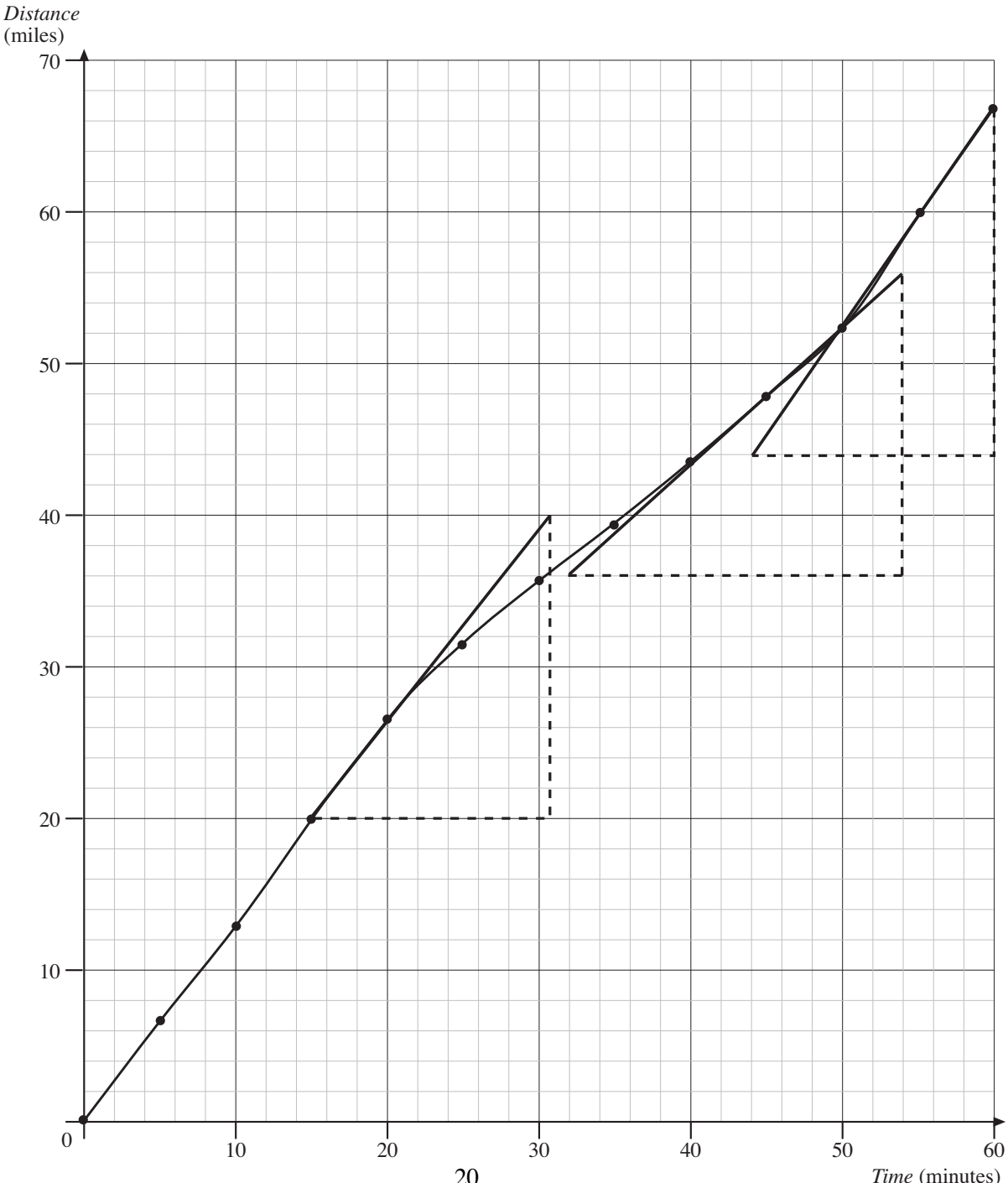
Rates of Change Answers

Instantaneous Speed

(a)



(i) At 20 mins, instantaneous speed $= \dfrac{20}{31-15} \approx 1.25$ miles per min $(\approx 75$ mph$)$

(ii) At 40 mins, instantaneous speed $= \dfrac{56-36}{54-32} \approx 0.91$ miles per min $(\approx 55$ mph$)$

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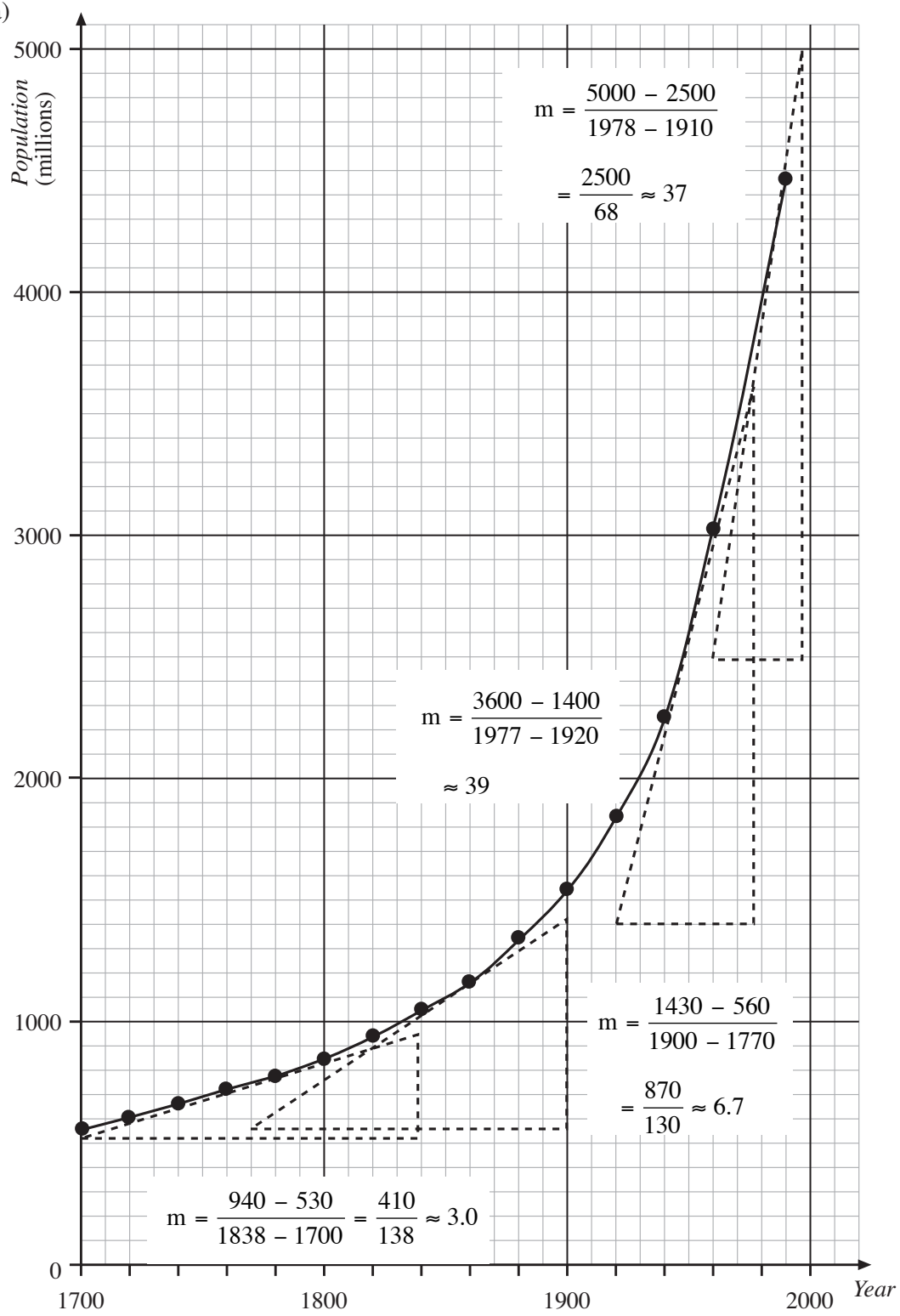
(iii) At 55 mins, instantaneous speed $= \dfrac{67 - 44}{60 - 44} = \dfrac{23}{16} \approx 1.4$ miles per min

$(\approx 85 \text{ mph})$

(b) Up to about 20 mins, the speed limit is being broken, and again from time 50 mins.

2. (a)



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1750 : about 3 million per year

1850 : about 7 million per year

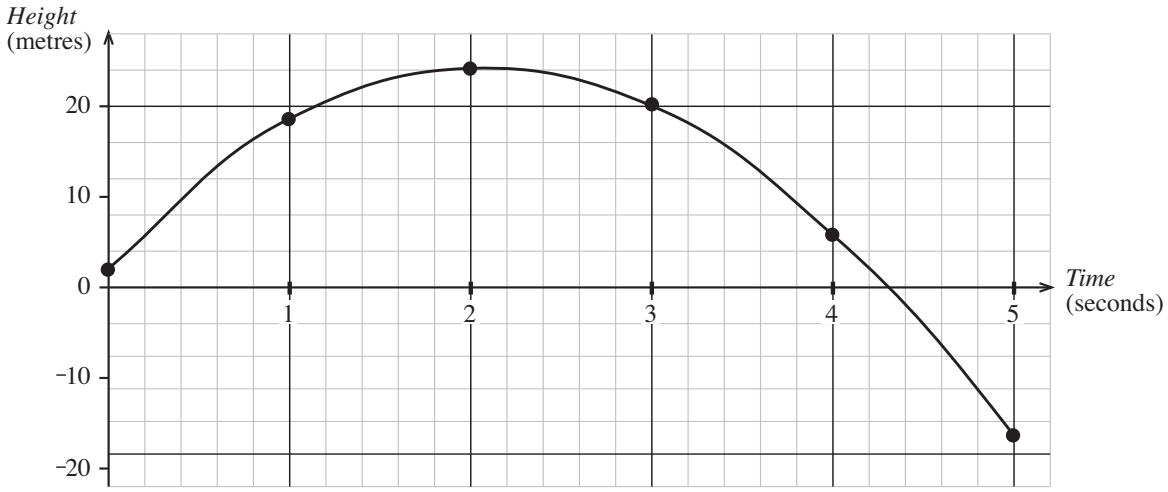
1950 : about 40 million per year

(b) The rate of increase of the world's populations is increasing so that the population is growing in an exponential way.

(c) In 1980, the gradient is about 37 million per year so the population estimate for 1981 is $4480 + 37 = 4517$ million.

3. (a)

Time, t (seconds)	0	1	2	3	4	5
Height, h (metres)	2	16	24	20	6	−18



(b) Velocity estimates $10,\ 0,\ -10,\ -20$ (m/s)

(c) Estimate is $18 + 10 \times 0.1 = 19$, actual value = 19.05

3 Finding the Gradient

1. (a) $\left(1+h, (1+h)^3\right)$ (b) $\dfrac{(1+h)^3 - 1}{h} = 3 + 3h + h^2$

(c) gradient $\to 3$ (d) gradient $= 3$

2. (a) $\left(2+h, (2+h)^3\right)$ (b) $\dfrac{(2+h)^3 - 2^3}{h} = 12 + 6h + h^2$

(c) gradient $\to 12$ (d) gradient $= 12$

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4 Gradient of Quadratics

1. (a) $4x$ (b) $2x+1$ (c) $2t+4$ (d) $2x-1$
   (e) $12\,l$ (f) $10-\frac{2x}{5}$ (g) $\frac{2y}{9}+3$ (h) $\frac{2n-5}{2}$
   (i) $1-12v$ (j) $\frac{3}{2}x+\frac{1}{5}$
2. (a) -7 (b) 5 (c) $23\frac{5}{6}$ (d) $\frac{9}{4}$ (e) 71
3. (a) $16\ \mathrm{m\,s^{-1}}$ (b) $-7\ \mathrm{m\,s^{-1}}$

5 Differentiation

1. (a) $3x^2+10x+3$ (b) $18t^2-20t+2$ (c) $10x-\frac{1}{x^2}$ (d) $3x^2-1$ (e) $\frac{3t^2+3}{5}$
2. (a) $2(x+2)$ (b) $3x^2-1$ (c) $3s^2+\frac{4}{3}s+\frac{1}{9}$
   (d) $\frac{8}{3}y^2+\frac{2}{3}y$ (e) $3x^2-5+\frac{1}{x^2}$
3. (a) 9 (b) $\frac{13}{4}$ (c) $(2, 19)$ (d) $(-1, 18)$
   (e) $(2, 41)$, $(1, 18)$
4. (a) $\frac{1}{3}$ (b) $\frac{3}{16}$ (feet per year)

6 Optimisation

1. $(-2, 55)$ max; $(6, -201)$ min
2. (a) $\left(\frac{3}{2}, \frac{5}{2}\right)$ min (b) $(3, 18)$ min, $(-3, -18)$ max
   (c) $(-7, -469)$ min, $(5, 395)$ max (d) $(2, 12)$ min
3. (a) $U = 5Q + 180 + \frac{12\,500}{Q}$ (b) 50, 680
4. (a) 47.6 mph (b) These are the usual speeds for a car.

7 Real Problems

1. $5\ \mathrm{m^2}$
2. (a) $13.33\ \mathrm{m^2}$ (b) $7.59\ \mathrm{m^2}$
3. 20 m
4. 120 m
5. (a) $V = hx^2$ (b) $15000 = 4xh + 2x^2$
   $\Rightarrow h = \frac{3750}{x} - \frac{x}{2}$

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(c) $V = \left(\frac{3750}{x} - \frac{x}{2}\right)x^2 = 3750x - \frac{x^3}{2}$

$\frac{dV}{dx} = 3750 - \frac{3x^2}{2} = 0 \Rightarrow x^2 = 2500 \Rightarrow x = 50$

and $V = 125000 \text{ cm}^3 = 125 \text{ litres}$

6. 120, 239, 159 mm

7. $S = 2\pi r^2 + 2\pi rh$ where $V = \pi r^2 h$

So $S = 2\pi r^2 + \frac{2V}{r}$

$\frac{dS}{dr} = 4\pi r - \frac{2V}{r^2} = 0 \Rightarrow r^3 = \frac{V}{2\pi} \Rightarrow r = \left(\frac{V}{2\pi}\right)^{\frac{1}{3}}$

$h = \frac{V}{\pi r^2} = \frac{V}{\pi}\left(\frac{V}{2\pi}\right)^{-\frac{2}{3}} = 2\left(\frac{V}{\pi}\right)^{\frac{1}{3}} \Rightarrow h = 2r \quad (= \text{diameter})$

8. (a) $v = 3t^2 - 24t + 45$

(b) $t = 3$ s or 5 s (c) $t = 4$ s

9. 3