PYTHAGORAS' THEOREM

Text

Contents

© University of Plymouth

Pythagoras' Theorem

1 Pythagoras' Theorem

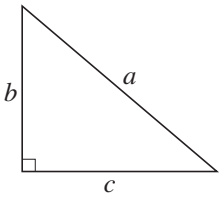
Pythagoras' Theorem gives a relationship between the lengths of the sides of a right angled triangle.

Pythagoras' Theorem states that:

> In any right angled triangle, the area of the square on the hypotenuse (the side opposite the right angle) is equal to the sum of the areas of the squares on the other two sides (the two sides that meet at the right angle).

For the triangle shown opossite,

$$a^2 = b^2 + c^2$$

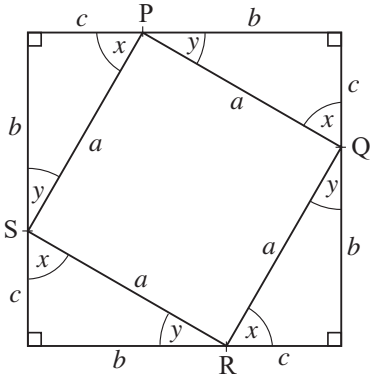


Note

The longest side of a right angled triangle is called the *hypotenuse*.

Proof

Draw a square of side $b + c$, as shown opposite.
Join up the points PQ, QR, RS, SP as shown, to give a quadrilateral, PQRS.



In fact, PQRS is a square as each side is equal to a (as the four triangles are congruent) and at the point P,

$$x + \text{angle SPQ} + y = 180°$$

But we know that $x + y = 90°$, so

$$\text{angle SPQ} = 90°$$

Similarly for the other three angles in PQRS. Thus PQRS is a square, and equating areas,

$$a^2 + 4 \times \left(\frac{1}{2}bc\right) = (b + c)^2$$

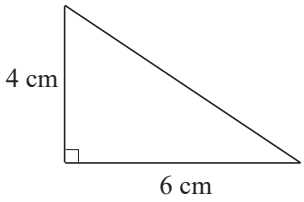
$$a^2 + 2bc = b^2 + 2bc + c^2$$

Hence $$a^2 = c^2 + b^2$$

© University of Plymouth

Worked Example 1

Find the length of the hypotenuse of the triangle shown in the diagram. Give your answer correct to 2 decimal places.



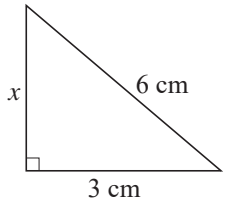
Solution

As this is a right angled triangle, Pythagoras' Theorem can be used. If the length of the hypotenuse is a, then $b = 4$ and $c = 6$.

So
$$a^2 = b^2 + c^2$$
$$a^2 = 4^2 + 6^2$$
$$a^2 = 16 + 36$$
$$a^2 = 52$$
$$a = \sqrt{52}$$
$$= 7.2 \text{ cm} \quad \text{(to one decimal place)}$$

Worked Example 2

Find the length of the side of the triangle marked x in the diagram.



Solution

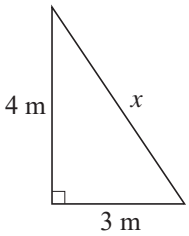
As this is a right angled triangle, Pythagoras' Theorem can be used. Here the length of the hypotenuse is 6 cm, so writing $a = 6$ cm and $c = 3$ cm with $b = x$, we have

$$a^2 = b^2 + c^2$$
$$6^2 = x^2 + 3^2$$
$$36 = x^2 + 9$$
$$36 - 9 = x^2$$
$$36 - 9 = x^2$$
$$27 = x^2$$
$$\sqrt{27} = x$$
$$x = 5.2 \text{ cm} \quad \text{(to one decimal place)}$$

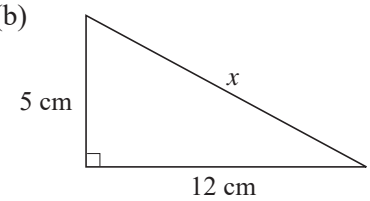
Exercises

1. Find the length of the side marked x in each triangle.

(a)

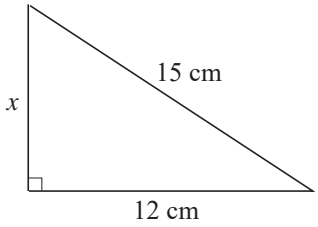


(b)

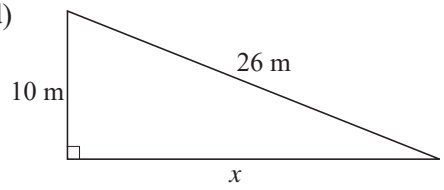


© University of Plymouth

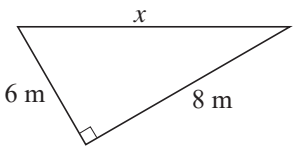
(c)



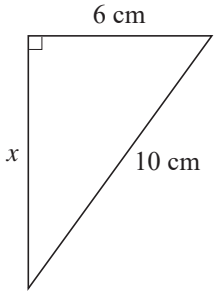
(d)



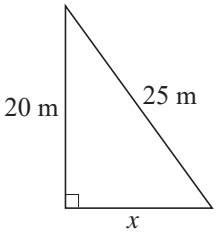
(e)



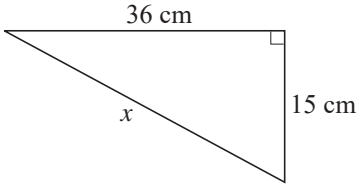
(f)



(g)

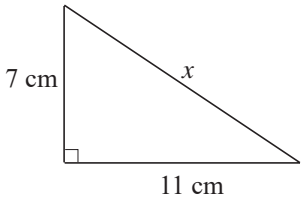


(h)

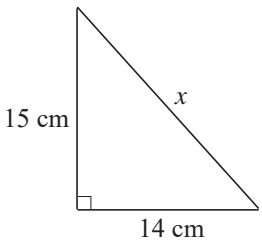


2. Find the length of the side marked x in each triangle. Give your answers correct to 2 decimal places.

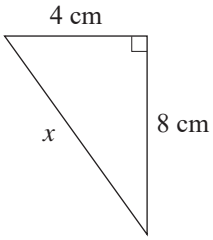
(a)



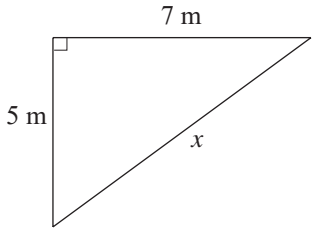
(b)



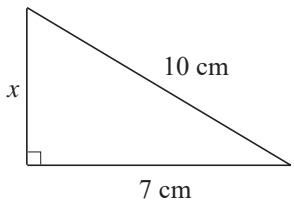
(c)



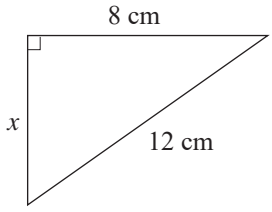
(d)



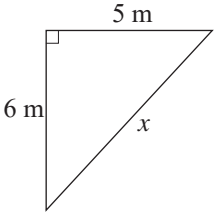
(e)



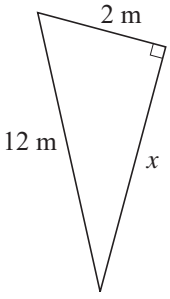
(f)



(g)

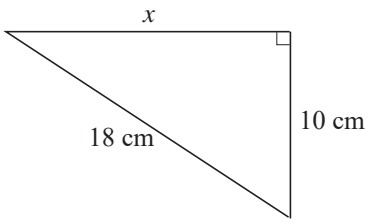


(h)

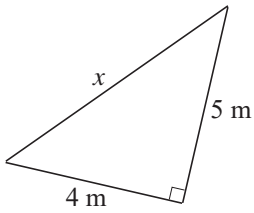


© University of Plymouth

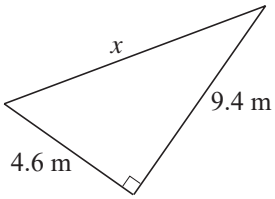
(i)



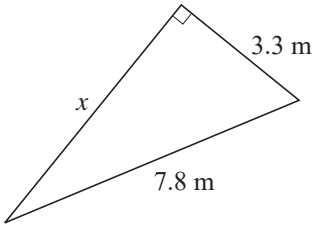
(j)



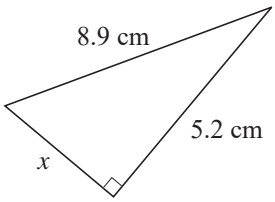
(k)



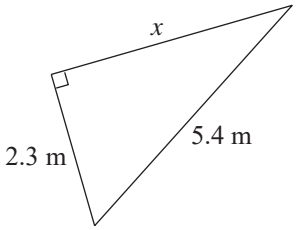
(l)



(m)



(n)

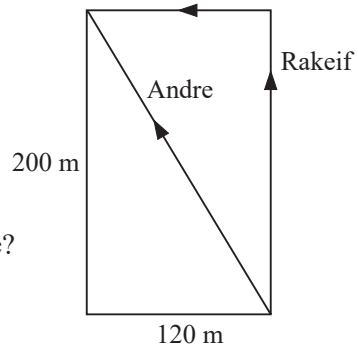


3. Andre runs diagonally across a school field, while Rakeif runs around the edge.

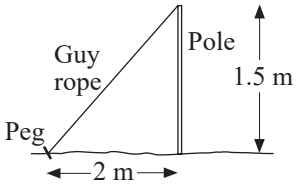
   (a) How far does Rakeif run?

   (b) How far does Andre run?

   (c) How much further does Rakeif run than Andre?



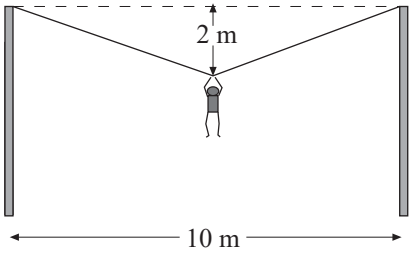
4. A guy rope is attached to the top of a tent pole, at a height of 1.5 metres above the ground, and to a tent peg 2 metres from the base of the pole. How long is the guy rope?



5. Daisy is 1.4 metres tall. At a certain time her shadow is 2 metres long. What is the distance from the top of her head to the tip of her shadow?

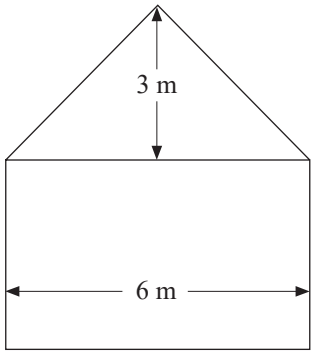
6. A rope of length 10 metres is stretched from the top of a pole 3 metres high until it reaches ground level. How far is the end of the line from the base of the pole?

7. A rope is fixed between two trees that are 10 metres apart. When a child hangs on to the centre of the rope, it sags so that the centre is 2 metres below the level of the ends. Find the length of the rope.

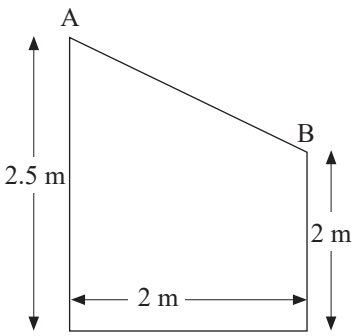


© University of Plymouth

8. The roof on a house that is 6 metres wide peaks at a height of 3 metres above the top of the walls. Find the length of the sloping side of the roof.



9. The picture shows a shed. Find the length, AB, of the roof.

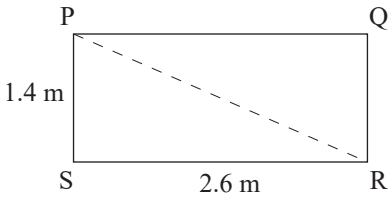


10. Rohan walks 3 km east and then 10 km north.

    (a) How far is he from his starting point?

    (b) He then walks east until he is 20 km from his starting point. How much further east has he walked

11. Jodieis building a shed. The base PQRS of the shed should be a rectangle measuring 2.6 metres by 1.4 metres.



    To check that or if the base is rectangular, Jodie has to measure the diagonal PR.

    (a) Calculate the length of PR when the base is rectangular. You *must* show all your working.

    (b) When building the shed Jodie finds angle $\mathrm{PSR} > 90°$.
    She measures PR. Which of the following statements is *true*?

    X: PR is greater than it should be.

    Y: PR is less than it should be.

    Z: PR is the right length.

Information

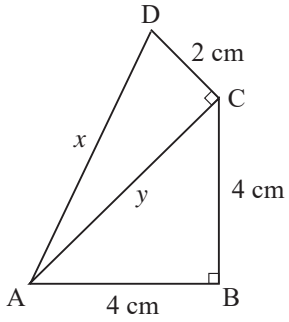
The Greeks, *(in their analysis of the arcs of circles) were the first to establish the relationships or ratios between the sides and the angles of a right angled triangle.*
The Chinese *also recognised the ratios of sides in a right angled triangle and some survey problems involving such ratios were quoted in Zhou Bi Suan Jing.*
It is interesting to note that sound waves are related to the sine curve. This discovery by Joseph Fourier, *a French mathematician, is the essence of the electronic musical instrument developments today.*

© University of Plymouth

2 Further Work with Pythagoras' Theorem

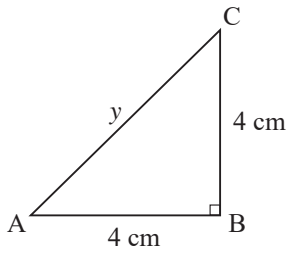
Worked Example 1

Find the length of the side marked x in the diagram.



Solution

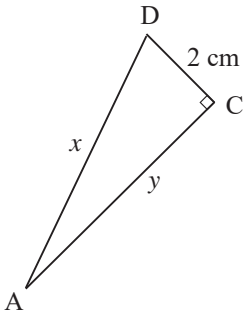
First consider triangle ABC. The unknown length of the hypotenuse has been marked y.



$$y^2 = (4\text{ cm})^2 + (4\text{ cm})^2$$
$$y^2 = 16\text{ cm}^2 + 16\text{ cm}^2$$
$$y^2 = 32\text{ cm}^2$$

Triangle ACD can now be considered, using the value for y^2.

From the triangle, $x^2 = y^2 + 2^2$, and using $y^2 = 32\text{ cm}^2$ gives



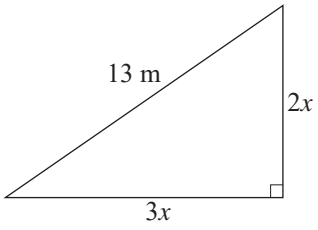
$$x^2 = 32\text{ cm}^2 + 4\text{ cm}^2$$
$$x^2 = 36\text{ cm}^2$$
$$x = \sqrt{36}\text{ cm}^2$$
$$x = 6\text{ cm}$$

Note

When finding the side x, it is not necessary to find $\sqrt{32}$, but to simply use $y^2 = 32\text{ cm}^2$.

Worked Example 2

Find the value of x as shown on the diagram.



Solution

Using Pythagoras' Theorem gives

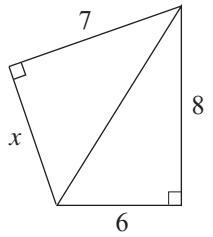
$$13^2 = (2x)^2 + (3x)^2$$
$$169 = 4x^2 + 9x^2 \qquad \text{(since } (2x)^2 = 2^2x^2 = 4x^2\text{)}$$
$$169 = 13x^2$$
$$13 = x^2$$
$$x = \sqrt{13}$$
$$= 3.61\text{ m} \qquad \text{(to 2 decimal places)}$$

© University of Plymouth

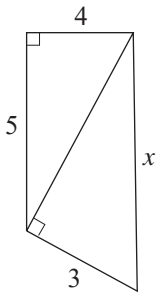
Exercises

1. Find the length of the side marked x in each diagram.

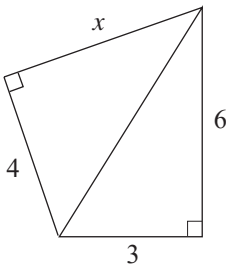
(a)



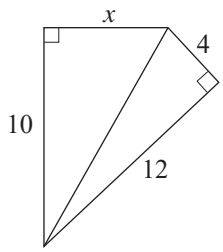
(b)



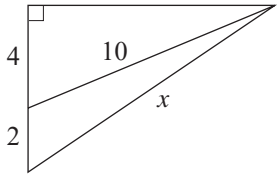
(c)



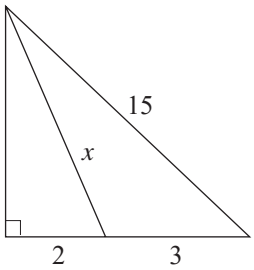
(d)



(e)

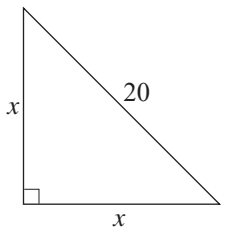


(f)

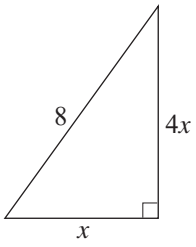


2. Find the length of the side marked x in the following situations.

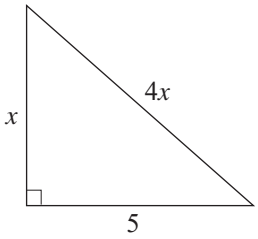
(a)



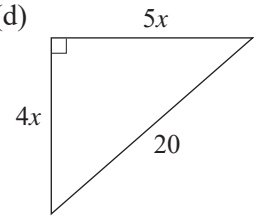
(b)



(c)

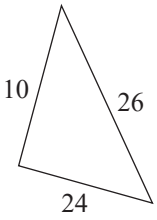


(d)

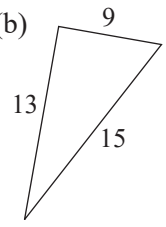


3. Which of the following triangles are right angled triangles?

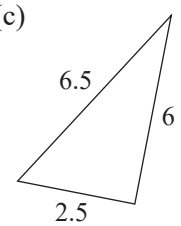
(a)



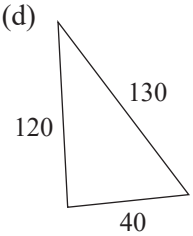
(b)



(c)

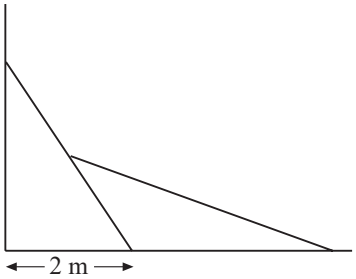


(d)



© University of Plymouth

4. A ladder of length 4 metres leans against a vertical wall. The foot of the ladder is 2 metres from the wall. A plank that has a length of 5 metres rests on the ladder, so that one end is halfway up the ladder.

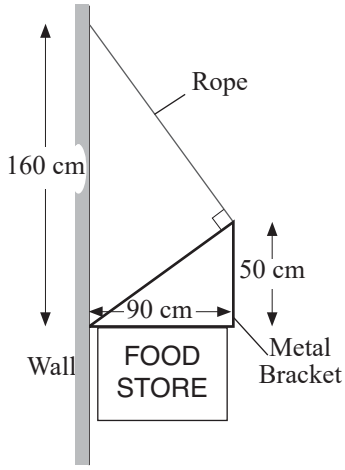


(a) How high is the top of the ladder?

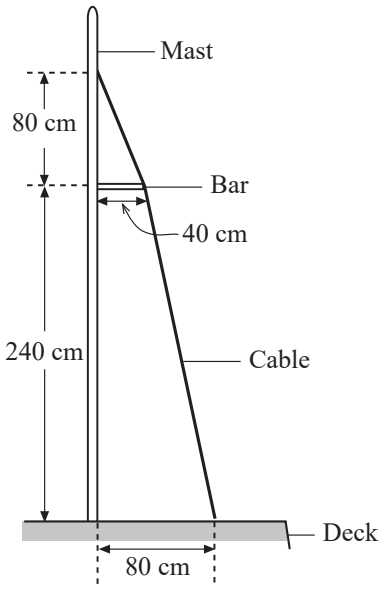
(b) How high is the top of the plank?

(c) How far is the bottom of the plank from the wall?

5. The diagram shows how the sign that hangs over a food store is suspended by a rope and a triangular metal bracket. Find the length of the rope.



6. The diagram shows how a cable is attached to the mast of a sailing dingy. A bar pushes the cable out away from the mast. Find the total length of the cable.



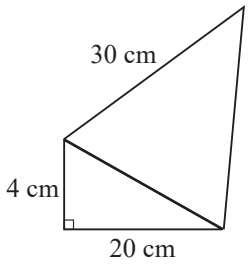
7. A helicopter flies in a straight line until it reaches a point 20 km east and 15 km north of its starting point. It then turns through $90°$ and travels a further 10 km.

(a) How far is the helicopter from its starting point?

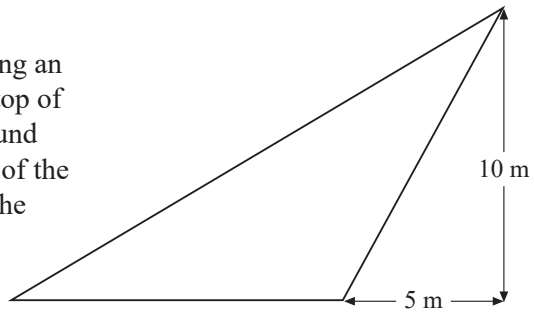
(b) If the helicopter turned $90°$ the other way, how far would it end up from its starting point?

© University of Plymouth

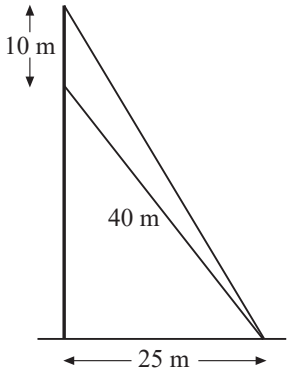
8. A cone is placed on a wedge. The dimensions of the wedge are shown in the diagram. The cone has a slant height of 30 cm. Find the height of the cone.



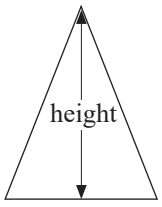
9. A simple crane is to be constructed using an isosceles triangular metal frame. The top of the frame is to be 10 metres above ground level and 5 metres away from the base of the crane, as shown in the diagram. Find the length of each side of the triangle.



10. A thin steel tower is supported on one side by two cables. Find the height of the tower and the length of the longer cable.



11. An isosceles triangle has two sides of length 8 cm and one of length 4 cm. Find the height of the triangle and its area.



12. Find the area of each the equilateral triangles that have sides of lengths
    (a) 8 cm
    (b) 20 cm
    (c) 2 cm

© University of Plymouth