Probability and Binomial Distributions Essential information

- ***Random variables*** can be discrete (fixed number of numerical values) or continuous (over a range of values)

- Note that for any discrete random variable, X,

$$\sum_{all\,x} P(X = x) = 1$$

- The ***expectation*** of a discrete random variable, x, is given by

$$E(X) = \sum_{all\,x} x \times P(X = x)$$

- The ***variance*** of a discrete random variable, X, is given by

$$V(X) = E(X^2) - [E(X)]^2$$

- A ***uniform distribution*** is when all outcomes for the random variable X are equally likely. If there are n possible outcomes, then

$$P(X = x) = \frac{1}{n} \quad , \quad x = 1, 2, ..., n$$

and $$E(X) = \frac{n+1}{2} \quad , \quad V(X) = \frac{n^2 - 1}{12}$$

- The ***Binomial distribution*** can be applied to an experiment with a finite number of trials, n, when the underlying probability of success, p, remains the same. We write

$$X \sim \text{Bin}(n, p) \text{ or } X \sim B(n, p)$$

- If $X \sim \text{Bin}(n, p)$, then

$$p(x = r) = \binom{n}{r} p^r q^{n-r}$$

where $q = 1 - p$ and $r = 0, 1, 2, ..., n$

- $\binom{n}{r} = \dfrac{n!}{r!\,(n-r)!}$

- $\binom{n}{0} = 1$, $\binom{n}{1} = n$, $\binom{n}{2} = \frac{n(n-1)}{2}$, etc and $\binom{n}{n} = 1$