Probability and Binomial Distributions Answers

1 Expectation

1. (a) discrete (b) continuous (c) discrete (d) discrete

2. (a)

Score	2	3	4	5	6	7	8	9	10	11	12
P	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

(b) 7

3. (a)

X	0	1	2	3
P	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

(b) (c) 1.5

4. (a) $c = \frac{1}{20}$ (c) $E(X) = 2.3$

5. $x = \frac{1}{6}$, $y = \frac{1}{12}$

2 Variance

1. $V(X) = \frac{35}{12}$, $s \approx 1.71$

2. (a) $V(X) = \frac{35}{6}$, $s \approx 2.42$

(b) $V(X) = \frac{3}{4}$, $s \approx 0.866$

3.

X	0	1	2	3
P	$\frac{1}{35}$	$\frac{12}{35}$	$\frac{18}{35}$	$\frac{4}{35}$

3 Probability Distributions

1. (a) 2 (b) 1

2. (a) $P(S = s) = \frac{1}{10}$ for $s = 1, 2, \ldots, 10$ (b) $\frac{11}{2}$ (c) $\sqrt{\frac{33}{4}} \approx 2.87$

3. (a) $E(X) = 1$; $V(X) = 1$ (b) $E(Y) = 0$; $V(Y) = 1.2$

4. (a)

X	2	3	4	5	6
P	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$

(b) $\frac{7}{12}$ (c) $\frac{14}{3}$, $\frac{10}{9}$

4 The Uniform Distribution

1. $E(X) = \frac{9}{2}$, $V(X) = \frac{63}{12}$

2. Yes

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5 Finding the Distribution

1. (a) 0.329 (b) 0.351 (c) 0.912
2. (a) 0.000138 (b) 0.0282 (c) 0.953
3. 1 $\left(pP(\mathrm{R}=1)=0.422\right)$
4. 0..230
5. 0.984
6. $\binom{8}{2}=\frac{8!}{2!\,6!}=28$ and $\left(\binom{8}{6}=\frac{8!}{6!\,2!}\right)=28$
7. (a) (i) 0.5289 (ii) 0.3158
   (b) 0.9699
8. (a) No; more than 2 outcomes (b) Yes
   (c) No; probability changes each time
9. (a) 4.8, 0.98
   (b) $X\sim B(6,0.8)$; $p(X\geq 5) = p(x=5)+p(x=6)$
   $= {}^{6}C_{5}(0.8)^{5}(0.2)+(0.8)^{6}$
   $= 0.655$ to 3 d.p.
   (c) 0.737 (d) 0.388

6 The Mean and Variance of the Binomial Distribution

1. 1. 0.263
2. 7
3. $\frac{2}{3}$
4. 0.51, 38
5. (a) B $(20,\ 0.2)$ (b) 4; 3.2 (c) 0.0026
7. B $(18,\ 0.8)$, mean value $= 18\times 0.8 = 14.4$, but
   $p(x=15)=0.2297$
   and $p(x=14)=0.2153$;
   So the most likely value is 15.

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8. (a) $(0.5)^{10} = 0.000977$

(b) In the first 10 steps, there need to be 9 in the dock direction and one in the reverse direction; this has

$$\text{probability} = {}^{10}\text{C}_9 (0.5)^9 (0.5)$$

$$= 10 \times (0.5)^{10}$$

The next 2 steps must both be in the dock direction, with

$$\text{probability} = (0.5)^2$$

Combining these two probabilities,

$$p(\text{fall into dock on } 12^{\text{th}} \text{ step}) = \left(10 \times (0.5)^{10}\right) \times (0.5)^2$$

$$= 10 \times (0.5)^{12}$$

$$\approx 0.00244$$

(c) 0.377

9. 0.2051

10. $P(\text{Don scores at least 3}) = 0.3222$

$P(\text{Yvette scores at least 3}) = 0.3174$