

# LINEAR EQUATIONS

## Text

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# Linear Equations

## 1 Simplifying Expressions

When simplifying expressions you should group terms which contain the same variable.



### Note

$x$  and  $x^2$  must be treated as if they were different letters. You cannot add an  $x$  term to an  $x^2$  term. The  $+$  and  $-$  signs go with the term which follows.



### Worked Example 1

Simplify each expression below.

- (a)  $4a + 3a + 6 + 2$                       (b)  $4a + 8b - 2a + 3b$   
 (c)  $x^2 + 5x - 8x + x^2 - 4$               (d)  $8x + y - 4x - 6y$



### Solution

- (a) The terms which involve  $a$  can be grouped. Also the 6 and 2 can be added.

$$4a + 3a + 6 + 2 = 7a + 8$$

- (b) The terms involving  $a$  are considered together, and then the terms involving  $b$ .

$$\begin{aligned} 4a + 8b - 2a + 3b &= 4a - 2a + 8b + 3b \\ &= 2a + 11b \end{aligned}$$

- (c) Here the  $x$  and  $x^2$  must be treated as if they are different letters.

$$\begin{aligned} x^2 + 5x - 8x + x^2 - 4 &= x^2 + x^2 + 5x - 8x - 4 \\ &= 2x^2 - 3x - 4 \end{aligned}$$

- (d) The different letters,  $x$  and  $y$ , must be considered in turn.

$$\begin{aligned} 8x + y - 4x - 6y &= 8x - 4x + y - 6y \\ &= 4x - 5y \end{aligned}$$

When a bracket is to be multiplied by a number or a letter, every term inside the bracket must be multiplied.



### Worked Example 2

Remove the brackets from each expression below.

- (a)  $6(x + 5)$               (b)  $3(2x + 7)$               (c)  $4(x - 3)$               (d)  $x(x - 4)$



## Solution

$$\begin{aligned} \text{(a)} \quad 6(x + 5) &= 6 \times x + 6 \times 5 \\ &= 6x + 30 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 3(2x + 7) &= 3 \times 2x + 3 \times 7 \\ &= 6x + 21 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad 4(x - 3) &= 4 \times x - 4 \times 3 \\ &= 4x - 12 \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad x(x - 4) &= x \times x - x \times 4 \\ &= x^2 - 4x \end{aligned}$$



## Exercises

1. Simplify each of these expressions.

$$\text{(a)} \quad a + 2a + 3a$$

$$\text{(b)} \quad 3a + 2 + 4 + 6$$

$$\text{(c)} \quad 3a + 2b + 8a + 4b$$

$$\text{(d)} \quad 4x + 2y + 8y + y$$

$$\text{(e)} \quad 5x + 2y + 8x - 3y$$

$$\text{(f)} \quad 6a + 7b + 3b - 4a$$

$$\text{(g)} \quad 4 + 6a - 3a + 2 + b$$

$$\text{(h)} \quad p + q + 2p - 8q + 3p$$

$$\text{(i)} \quad x + y - 8x + 2y$$

$$\text{(j)} \quad 4x - 3p + 2p - 2x$$

$$\text{(k)} \quad 7x - 4z + 8x - 5z$$

$$\text{(l)} \quad 3z - 4x + 2z - 10x$$

$$\text{(m)} \quad 3q - 4x + 8a - 2x + q$$

$$\text{(n)} \quad x + y + z - p - q - y$$

$$\text{(o)} \quad x + 6 + y + 4 + 2x - 3y$$

$$\text{(p)} \quad 4x - 8q + 17x - 24q$$

$$\text{(q)} \quad -x + y + x + y$$

$$\text{(r)} \quad 4x + 7y - 3x - 8y + x + y$$

$$\text{(s)} \quad -8x + 7y - 11x + 4y$$

$$\text{(t)} \quad 6x - 18y + 17x - 4$$

$$\text{(u)} \quad x + y - 8x - 11y$$

$$\text{(v)} \quad 4p + 8q - 8p - 4q$$

2. Simplify each of the following expressions.

$$\text{(a)} \quad 2x^2 + 3x + 4x^2 + 5x$$

$$\text{(b)} \quad x^2 + 8x + 5x + 10$$

$$\text{(c)} \quad x^2 + 6x + 4x + x^2$$

$$\text{(d)} \quad x^2 + x + 10 + x + 4x^2$$

$$\text{(e)} \quad 5x^2 - x - 6x^2 + 8x$$

$$\text{(f)} \quad 4x^2 - 3y^2 - x^2 + y^2$$

$$\text{(g)} \quad x^2 + y^2 - x - y + x^2$$

$$\text{(h)} \quad 4x^2 - 7x + 1 + x^2 + 4x - 11$$

$$\text{(i)} \quad x^2 - y^2 - x - y + 2x^2 - 2y^2$$

$$\text{(j)} \quad y^2 + y - 4 + y + 4y^2$$

$$\text{(k)} \quad ab + cd + 4ab$$

$$\text{(l)} \quad xy + xz + xy + 4xz$$

$$\text{(m)} \quad 4ab + 7ab - 3ad$$

$$\text{(n)} \quad 4pq - 3qr + 5pq$$

3. Remove the brackets from each expression below.

- |                 |                  |                      |
|-----------------|------------------|----------------------|
| (a) $3(x + 5)$  | (b) $4(6 + x)$   | (c) $7(x + 2)$       |
| (d) $2(x + 6)$  | (e) $5(x + 2)$   | (f) $4(2x + 3)$      |
| (g) $5(3x + 2)$ | (h) $8(5x + 3)$  | (i) $7(x - 6)$       |
| (j) $8(5 - x)$  | (k) $4(2x - 7)$  | (l) $7(5x - 3)$      |
| (m) $6(3x - 5)$ | (n) $4(x - 2y)$  | (o) $5(x + 2y + 3z)$ |
| (p) $x(5 + x)$  | (q) $a(2 - a)$   | (r) $4(b - 3)$       |
| (s) $2x(x - 6)$ | (t) $4x(2x + 3)$ | (u) $3x(7 - 2x)$     |
| (v) $8x(x - 5)$ |                  |                      |

4. Simplify each of the following expressions, by first removing all the brackets.

- |                           |                             |
|---------------------------|-----------------------------|
| (a) $3(a + 2) + 4(a + 5)$ | (b) $2(2x + 4) + 3(x + 5)$  |
| (c) $5(x + 2) + 3(x + 4)$ | (d) $x(x + 1) + x(x + 6)$   |
| (e) $4(x + 1) + x(x + 5)$ | (f) $2(a + b) + 5(2a + 3b)$ |

5. Simplify

- |  |  |
|--|--|
| (a) $3(x - y) - 2(y - x)$                      | (b) $3(5x + 2) - 2(4x + 3)$                    |
| (c) $\frac{3x}{12} - \frac{x + 2}{8}$          | (d) $\frac{1}{4}(x - 8) + \frac{1}{2}(2x + 4)$ |
| (e) $\frac{5}{2}(4 - y) - \frac{3}{2}(2y + 8)$ | (f) $\frac{4y}{15} - \frac{-3(2 - 3y)}{8}$     |

6. Write as a single fraction in its simplest form

$$\frac{x + 2}{2} + \frac{x - 4}{3}$$

## 2 Simple Equations

To solve simple equations you must carry out the *same* operation (addition, subtraction, multiplication or division) on *both* sides of the equation so that the new equation is still balanced.



### Worked Example 1

Solve each of the following equations.

- |                 |                  |               |                       |
|-----------------|------------------|---------------|-----------------------|
| (a) $x + 3 = 8$ | (b) $x - 8 = 11$ | (c) $4x = 32$ | (d) $\frac{x}{6} = 7$ |
|-----------------|------------------|---------------|-----------------------|



## Solution

- (a) To solve this equation, subtract 3 from both sides.

$$\begin{aligned}x + 3 &= 8 \\x + 3 - 3 &= 8 - 3 \\x &= 5\end{aligned}$$

- (b) To solve this equation, add 8 to both sides.

$$\begin{aligned}x - 8 &= 11 \\x - 8 + 8 &= 11 + 8 \\x &= 19\end{aligned}$$

- (c) To solve this equation divide both sides by 4.

$$\begin{aligned}4x &= 32 \\ \frac{4x}{4} &= \frac{32}{4} \\ x &= 8\end{aligned}$$

- (d) To solve this equation multiply both sides by 6.

$$\begin{aligned}\frac{x}{6} &= 7 \\ \frac{x}{6} \times 6 &= 7 \times 6 \\ x &= 42\end{aligned}$$



## Worked Example 2

A packet of sweets is divided equally among 5 children and each child is given 4 sweets. Write down an equation to describe this situation and solve it to find the number of sweets in the packet.



## Solution

Let  $x$  be the number of sweets in the packet.

Then 
$$\frac{x}{5} = 4$$

since the 5 children have 4 sweets each. Now the equation can be solved by multiplying both sides by 5.

$$\begin{aligned}\frac{x}{5} \times 5 &= 4 \times 5 \\ x &= 20\end{aligned}$$



## Exercises

1. Solve each of these equations.

(a)  $x + 6 = 10$

(b)  $x - 7 = 3$

(c)  $x + 4 = 7$

(d)  $x - 1 = 11$

(e)  $x - 6 = 8$

(f)  $x + 5 = 3$

(g)  $5x = 45$

(h)  $6x = 24$

(i)  $6x = 108$

(j)  $7x = 56$

(k)  $3x = 102$

(l)  $6x = 42$

(m)  $\frac{x}{2} = 5$

(n)  $\frac{x}{2} = 12$

(o)  $\frac{x}{5} = 10$

(p)  $\frac{x}{3} = 4$

(q)  $\frac{x}{7} = 3$

(r)  $\frac{x}{6} = 11$

(s)  $x - 5 = 3$

(t)  $x + 8 = 14$

(u)  $4x = 104$

(v)  $\frac{x}{2} = 18$

(w)  $x - 3 = 12$

(x)  $x + 7 = 11$

2. Solve each of these equations.

(a)  $x + 6 = 2$

(b)  $x + 8 = 3$

(c)  $x - 5 = -2$

(d)  $x + 2 = -4$

(e)  $x - 2 = -6$

(f)  $x + 4 = -10$

(g)  $2x = -12$

(h)  $3x = -24$

(i)  $5x = -60$

(j)  $\frac{x}{2} = -8$

(k)  $x - 2 = -5$

(l)  $x + 6 = -14$

(m)  $x - 10 = -2$

(n)  $x - 12 = -4$

(o)  $x - 7 = -1$

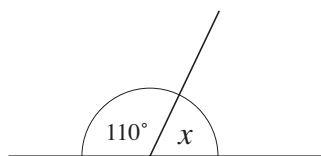
(p)  $x + 4 = -1$

(q)  $x + 12 = 2$

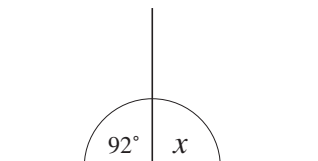
(r)  $x + 10 = -16$

3. The angles on a straight line add up to  $180^\circ$ . Write down and solve an equation for each diagram shown below.

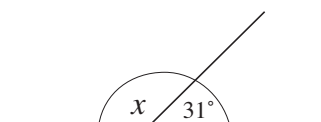
(a)



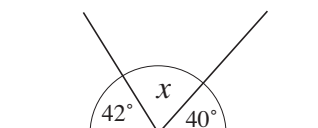
(b)



(c)



(d)



4. Arthur is two years older than his brother. His brother is 16. Write down an equation which uses  $x$  to represent Arthur's age. Solve the equation for  $x$ .

5. To pay for a school trip, 12 students take the same amount of money to school. If the total money collected is £54 and the amount each student takes is  $x$ , write down an equation to describe this situation. Solve your equation for  $x$ .
6. The cost of a journey increases by £3 to £41. If  $x$  is the original cost, write down an equation to describe this situation. Solve your equation for  $x$ .
7. Melanie knows that when a certain number is doubled, the answer is 52. Explain in words, starting with 52, how she can work out the number.
8. Lennox thought of a number. He doubled his number. His answer was 24. What number did Lennox think of?
9. Grace is twice as old as Carla. Carla is 2 years younger than George. George is 11 years old.
  - (a) How old is Carla?
  - (b) How old is Grace?

## 3 Solving Linear Equations

Most equations require a number of steps to solve them. These steps must be logical so that the new equation still balances. Whatever you do to one side of an equation you *must* do the same to the other side. The following examples illustrate these steps.



### Worked Example 1

Solve the following equations.

- |                           |                    |
|---------------------------|--------------------|
| (a) $3x + 7 = 13$         | (b) $5x - 8 = 13$  |
| (c) $\frac{x}{5} - 2 = 3$ | (d) $4(x - 3) = 8$ |



### Solution

- (a) First subtract 7 from both sides of the equation.

$$\begin{aligned}
 3x + 7 &= 13 \\
 3x + 7 - 7 &= 13 - 7 \\
 3x &= 6
 \end{aligned}$$

Next divide both sides of the equation by 3.

$$\begin{aligned}
 \frac{3x}{3} &= \frac{6}{3} \\
 x &= 2
 \end{aligned}$$

- (b) First add 8 to both sides of the equation.

$$5x - 8 = 13$$

$$5x - 8 + 8 = 13 + 8$$

$$5x = 21.$$

Then divide both sides of the equation by 5.

$$\frac{5x}{5} = \frac{21}{5}$$

$$x = \frac{21}{5}$$

$$= 4\frac{1}{5}$$

- (c) First add 2 to both sides of the equation.

$$\frac{x}{5} - 2 = 3$$

$$\frac{x}{5} - 2 + 2 = 3 + 2$$

$$\frac{x}{5} = 5$$

Then multiply both sides of the equation by 5.

$$\frac{x}{5} \times 5 = 5 \times 5$$

$$x = 25$$

- (d) First remove the brackets, multiplying each term inside the bracket by 4.

$$4(x - 3) = 8$$

$$4x - 12 = 8$$

Then add 12 to both sides of the equation.

$$4x - 12 + 12 = 8 + 12$$

$$4x = 20$$

Finally divide both sides by 4.

$$\frac{4x}{4} = \frac{20}{4}$$

$$x = 5$$

Sometimes equations may contain the letter  $x$  on both sides of the equation or a  $-x$  term. The following examples show how to deal with these cases.





## Worked Example 2

Solve these equations.

(a)  $4x + 6 = 3x + 10$       (b)  $6 - 2x = 8$       (c)  $4x - 2 = 8 - 6x$



## Solution

(a) As  $x$  appears on both sides of the equation, first subtract  $3x$  from both sides.

$$\begin{aligned} 4x + 6 &= 3x + 10 \\ 4x + 6 - 3x &= 3x + 10 - 3x \\ x + 6 &= 10 \end{aligned}$$

Then subtract 6 from both sides.

$$\begin{aligned} x + 6 - 6 &= 10 - 6 \\ x &= 4 \end{aligned}$$

(b) As the left-hand side contains  $-2x$ , add  $2x$  to both sides.

$$\begin{aligned} 6 - 2x &= 8 \\ 6 - 2x + 2x &= 8 + 2x \\ 6 &= 8 + 2x \end{aligned}$$

Then subtract 8 from both sides.

$$\begin{aligned} 6 - 8 &= 8 + 2x - 8 \\ -2 &= 2x \end{aligned}$$

Finally divide both sides by 2.

$$\begin{array}{r} \cancel{-2}^{-1} \\ \hline \cancel{2}_1 \end{array} = \frac{\cancel{2x}^1}{\cancel{2}_1} \\ -1 = x \text{ or } x = -1$$

(c) As one side contains  $-6x$ , add  $6x$  to both sides.

$$\begin{aligned} 4x - 2 &= 8 - 6x \\ 4x - 2 + 6x &= 8 - 6x + 6x \\ 10x - 2 &= 8 \end{aligned}$$

Then add 2 to both sides of the equation.

$$\begin{aligned} 10x - 2 + 2 &= 8 + 2 \\ 10x &= 10 \end{aligned}$$

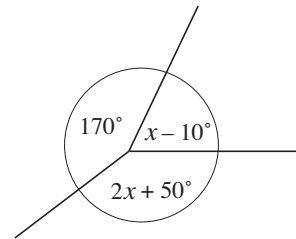
Finally divide both sides by 10.

$$\begin{array}{r} \cancel{10x}^1 \\ \hline \cancel{10}_1 \end{array} = \frac{\cancel{10}^1}{\cancel{10}_1} \\ x = 1$$



### Worked Example 3

Use the information in the diagram to write down an equation and then find the value of  $x$ .



### Solution

The three angles shown must add up to  $360^\circ$ , so

$$170 + 2x + 50 + x - 10 = 360$$

$$210 + 3x = 360$$

Subtracting 210 from both sides gives

$$210 + 3x - 210 = 360 - 210$$

$$3x = 150$$

Then dividing both sides by 3 gives

$$\frac{3x}{3} = \frac{150}{3}$$

$$x = 50$$



### Worked Example 4

Arianne, Jovan and Kerry were playing a card game.

Arianne scored  $x$  points.

Jovan scored 3 points fewer than Arianne.

Kerry scored twice as many points as Jovan.

Together they scored 39 points.

- Write down, in terms of  $x$ , an expression for the number of points scored by Kerry.
- Write an equation which may be used to find the value of  $x$ .
- How many points did Arianne score?



### Solution

(a) Number of points scored by Jovan =  $x - 3$

Number of points scored by Kerry =  $2(x - 3)$

(b) Total number of points =  $x + (x - 3) + 2(x - 3) = 39$

(c) Solving,  $4x - 9 = 39$

$$4x = 39 + 9$$

$$4x = 48$$

$$x = 12$$

So Arianne scored 12 points.



## Exercises

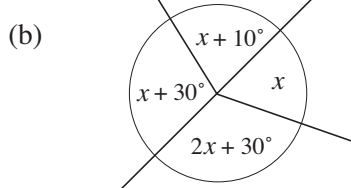
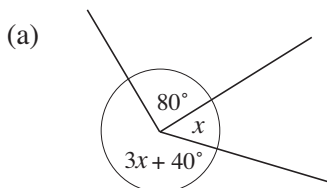
1. Solve each of these equations.

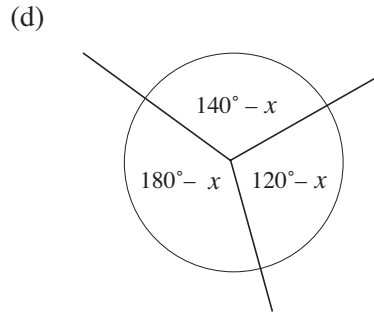
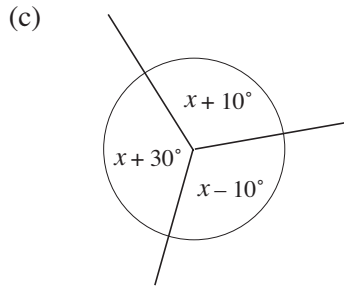
- |                           |                           |                            |
|---------------------------|---------------------------|----------------------------|
| (a) $3x + 6 = 48$         | (b) $5x - 6 = 39$         | (c) $2x - 6 = 22$          |
| (d) $6x - 7 = 41$         | (e) $8x - 3 = 29$         | (f) $6x + 12 = 20$         |
| (g) $4x + 18 = 2$         | (h) $5x + 10 = 5$         | (i) $3x + 6 = 1$           |
| (j) $5(x + 2) = 45$       | (k) $3(x - 2) = 12$       | (l) $2(x + 7) = 10$        |
| (m) $3(2x - 1) = 57$      | (n) $3(2x + 7) = 27$      | (o) $5(5x + 1) = 20$       |
| (p) $4(2x + 3) = -8$      | (q) $5(3x - 1) = -2$      | (r) $2(8x + 5) = -2$       |
| (s) $6x - 8 = -26$        | (t) $4(x + 15) = 60$      | (u) $5x - 8 = -10$         |
| (v) $\frac{x}{4} - 1 = 8$ | (w) $\frac{x}{3} + 2 = 7$ | (x) $\frac{2x}{5} + 1 = 3$ |

2. Solve these equations.

- |                            |   |
|----------------------------|---|
| (a) $2x + 6 = x + 3$       | (b) $4x - 8 = 5x - 2$                   |
| (c) $6x + 7 = 2x + 20$     | (d) $x + 6 = 2x - 8$                    |
| (e) $3x + 7 = 2x + 11$     | (f) $10x + 2 = 8x + 22$                 |
| (g) $6 - x = 5$            | (h) $2 - x = 5$                         |
| (i) $3 - x = -10$          | (j) $14 - 3x = 5$                       |
| (k) $10 - 2x = 2$          | (l) $4 - 3x = 2$                        |
| (m) $x + 2 = 8 - x$        | (n) $x + 4 = 10 - 2x$                   |
| (o) $x + 4 = 9 - 2x$       | (p) $8 - x = 12 - 2x$                   |
| (q) $22 - 4x = 18 - 2x$    | (r) $3 - 6x = 2 - 4x$                   |
| (s) $3(x + 2) = 5(x - 2)$  | (t) $4 = 8 - \frac{x}{3}$               |
| (u) $3 - \frac{x}{4} = -5$ | (v) $4(x - 2) = 3(x + 2)$               |
| (w) $5 = 18 - \frac{x}{3}$ | (x) $2 - \frac{x}{4} = 1 - \frac{x}{6}$ |

3. For each of the following diagrams, write down an equation involving  $x$  and solve it.





4. A rope of length 10 m is used to mark out a rectangle, so that the two long sides are 1 m longer than the short sides. If  $x$  is the length of the short sides, write down an equation to describe this situation and hence find  $x$ .
5. You ask a friend to think of a number, double it and add 10. His answer is 42. If  $x$  is the number your friend thought of, write down the relevant equation and find  $x$ .
6. Six teams enter a competition. There are  $x$  members in each team. If 8 people drop out and 34 complete the competition, write down an equation and solve it to find the number in each team at the start of the competition.
7. Pearl had £12 and Kurt had £2. Each of them received £ $x$  for a job. Write an expression in terms of  $x$  for the amount of money
- Pearl now has
  - Kurt now has.
- After they were paid for the job, Pearl's amount of money was 3 times as much as Kurt's.
- Write an equation in terms of  $x$  to represent the information given.
  - Solve the equation.
  - How much money was Pearl paid for the job?
8. A student was asked to think of a number and follow these instructions. In each case, let  $x$  be the number the student thinks of, write down an equation, and find the value of  $x$ .
- Think of a number, add 6 and double it. *Answer 18*
  - Think of a number, divide by 2 and add 10. *Answer 16*
  - Think of a number, divide by 2, add 2 and multiply by 2. *Answer 9*
  - Think of a number, subtract 7, divide by 2 and multiply by 10. *Answer 115*
9. Four consecutive numbers, when added together, give a total of 114. If  $x$  is the lowest number, write down an equation and solve it.
10. Adam thinks of a number. He doubles it and then adds 5. The answer is 17. What was his number?

11. (a) Write, in symbols, the rule,

*To find  $y$ , double  $x$  and add 1.*

- (b) Use your rule from part (a) to calculate the value of  $x$  when  $y = 9$ .

12. Clifton uses this rule,

*Start with a number, divide it by 2 and then add 3. Write down the result.*

- (a) What is the result when Clifton starts with 8?  
 (b) What number did Clifton start with when the result is 5?

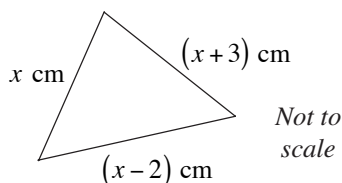
13. Solve the equation

$$11x + 5 = x + 25$$

14. Solve the equations

(a)  $4x - 1 = 17$                       (b)  $11y + 3 = 5y + 27$

15. The lengths of the sides of a triangle are  $x$  cm,  $(x + 3)$  cm and  $(x - 2)$  cm.



- (a) What is the perimeter of the triangle in terms of  $x$ ?  
 (b) The triangle has a perimeter of 22 cm.  
 (i) Write down an equation in  $x$ .  
 (ii) Use your equation to find the length of each side of the triangle.
16. A carton of orange juice costs  $x$  pence.  
 (a) Write, in terms of  $x$ , the cost of two cartons of orange juice.  
 The cost of a carton of pineapple juice is 20 pence more than the cost of a carton of orange juice.  
 (b) Write in terms of  $x$ , the cost of a carton of pineapple juice.  
 Sam pays £3.40 for three cartons of orange juice and two cartons of pineapple juice. He writes down the correct equation  

$$3x + 2(x + 20) = 340$$
  
 (c) Solve this equation to find the cost of a carton of orange juice.
17. Solve  $8x - 2(3x - 8) = 24$ .