GRAPHS

Text

Contents

Graphs

1 Solution of Simultaneous Equations by Graphs

Solutions to pairs of simultaneous equations can be found by plotting lines and finding the coordinates of the point where they intersect. They can be solved analytically but this introduces a more general method which we extend in later units.

Worked Example 1

Solve the pair of simultaneous equations

$$x + y = 8 \quad \text{and} \quad 2x + 3y = 21$$

Solution

First it can be helpful to write the two equations in the form $y = \ldots$.

For the first equation,

$$x + y = 8$$

$$y = 8 - x \qquad \text{(subtracting } x\text{)}$$

For the second equation,

$$2x + 3y = 21$$

$$3y = 21 - 2x \qquad \text{(subtracting } 2x\text{)}$$

$$y = 7 - \frac{2}{3}x \qquad \text{(dividing by 3)}$$

Now two pairs of coordinates can be found for each line.

For $y = 8 - x$

If $x = 0$, $y = 8$ so $(0, 8)$ lies on the line

If $x = 8$, $y = 8 - 8$

$= 0$ so $(8, 0)$ lies on the line.

For $y = 7 - \frac{2}{3}x$

If $x = 0$, $y = 7 - 0$

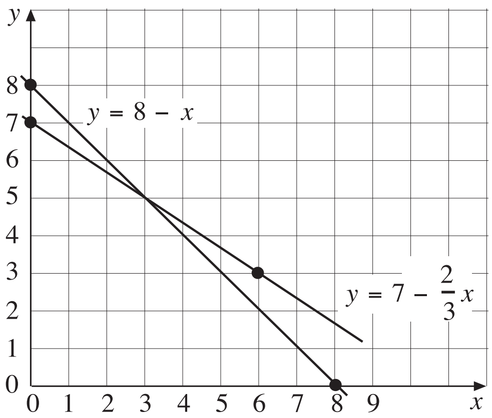
$= 7$ so $(0, 7)$ lies on the line.

If $\quad x = 6 \quad , \quad y = 7 - \frac{2}{3} \times 6$

$= 7 - 4$

$= 3$ so $(6, 3)$ lies on the line.

These points are then used to plot the lines shown below.



The two lines intersect at the point (3, 5), so the solution is

$$x = 3 \text{ and } y = 5$$

Note

We can easily solve these equations analytically by writing

$$x + y = 8 \quad \Rightarrow \quad \left.\begin{array}{l} 2x + 2y = 16 \quad (1) \\ 2x + 3y = 21 \quad (2) \end{array}\right\} \text{ subtract } (2) \text{ from } (1) \text{ to give } y = 5$$

Substituting back in either equation gives x; for example,

$$x = 8 - y = 8 - 5 = 3$$

Hence solution at (3, 5).

Exercises

1. Use the graph below to solve the simultaneous equations.

(a) $y = 8 - x$

$y = 2x - 1$

(b) $y = 8 - x$

$y = 3x - 20$

(c) $y = 2x - 1$

$y = -\frac{1}{6}x - 1$

(d) $y = 3x - 20$

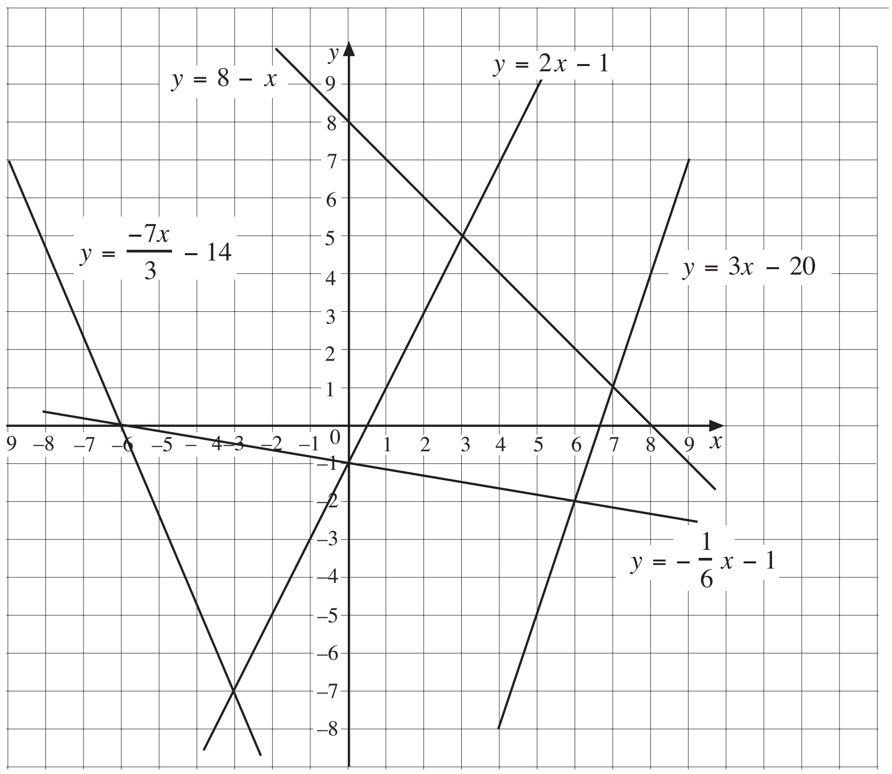
$y = -\frac{1}{6}x - 1$

(e) $y = -\frac{7}{3}x - 14$

$y = 2x - 1$

(f) $y = -\frac{1}{6}x - 1$

$y = -\frac{7}{3}x - 14$



2. (a) Draw a set of axes with x-values from -4 to 4 and y-values from -5 to 5.

   (b) Write down the coordinates of three points on the line $y = x - 1$ and use them to draw the line $y = x - 1$.

   (c) On the same set of axes draw the lines $y = 5 - x$ and $y = 3x + 1$.

   (d) Write down the solution of each set of simultaneous equations.

   (i) $y = x - 1$
   $y = 5 - x$

   (ii) $y = x - 1$
   $y = 3x + 1$

   (iii) $y = 5 - x$
   $y = 3x + 1$

3. (a) Write the equations $y - 3x = 1$ and $y + 2x = 6$ in the form $y = \ldots.$

   (b) Draw the graphs of both equations.

   (c) What is the solution of the simultaneous equations?

4. A discount store sells CDs and DVDs. The price of every CD is £x and the price of every DVD is £y.

(a) Jane buys 2 CDs and 4 DVDs which cost a total of £40. Write down an equation involving x and y using this information.

(b) Write down an equation in the form $y = \ldots$.

(c) Christopher buys 3 CDs and 2 DVDs which cost a total of £36. Write down a second equation using this information.

(d) Write this equation in the form $y = \ldots$.

(e) Draw the graphs of both equations on the same set of axes.

(f) What is the price of a CD?

(g) What is the price of a DVD?

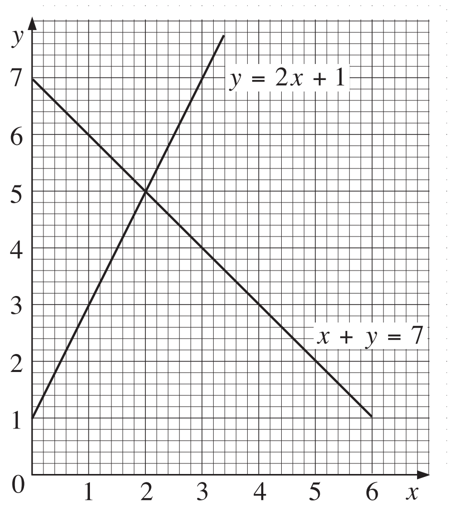
5. An enterprising schoolboy charges £2 to wash a car and £5 to wash and polish a car. One day he earns £80 and cleans 28 cars. Let x be the number of cars that he only washed, and y the number of cars washed and polished.

(a) Write down two equations involving x and y.

(b) Write these equations in the form $y = \ldots$.

(c) Draw the graphs of both equations and find out how many cars were washed and polished.

6.



The diagram shows the graphs of the equations

$$y = 2x + 1 \quad \text{and} \quad x + y = 7$$

Use the diagram to solve the simultaneous equations

$$y = 2x + 1$$
$$x + y = 7$$

7. A longlife battery and a standard battery were both tested for their length of life.

The longlife battery lasted for x hours.

The standard life battery lasted for y hours.

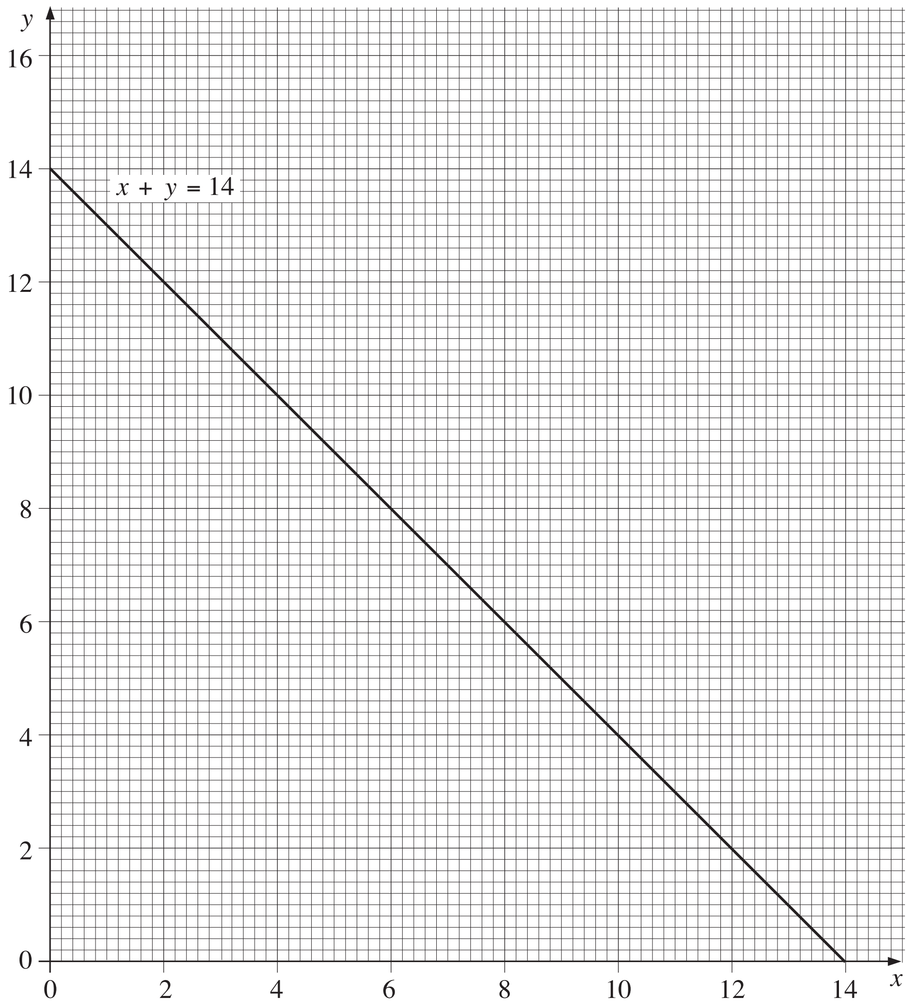
(a) The combined length of life of the two batteries was 14 hours.

Explain why $x + y = 14$

(b) The longlife battery lasted 3 hours longer than the standard battery.

Write down another equation connecting x and y.

The graph of $x + y = 14$ has been drawn below.



(c) Complete a copy of the table of values for your equation in (b) and use it to draw the graph of your equation.

x	3	7	10
y			

(d) Use your graphs to find the length of life of each type of battery.

8. For off-peak electricity, customers can choose to pay by *Method A* or *Method B*.

Method A: £8 per quarter plus 4 p per unit.

Method B: £12 per quarter plus 3 p per unit.

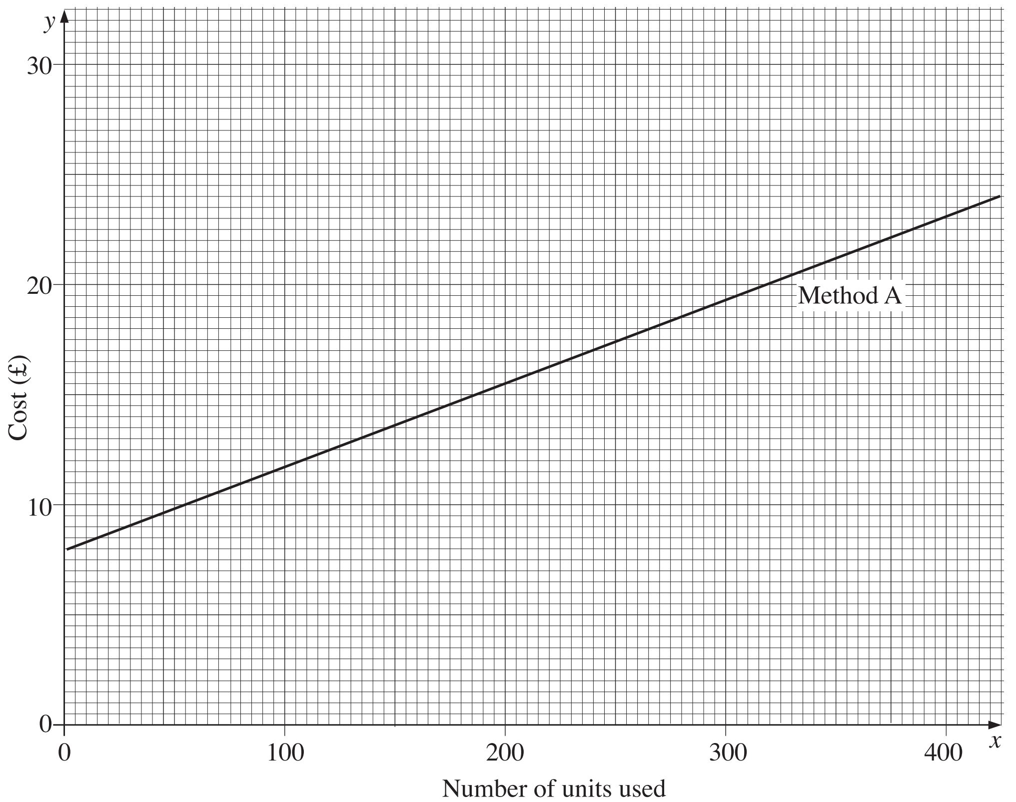
A customer uses x units. The quarterly cost is £y.

The cost, £y, by *Method B* is $y = 0.03x + 12$.

(a) (i) Complete the table of values of $y = 0.03x + 12$.

x	100	200	300
y	15		

(ii) Draw the graph of $y = 0.03x + 12$.



(b) For a certain number of units both methods give the same cost.
Use the graphs to find

(i) this number of units,

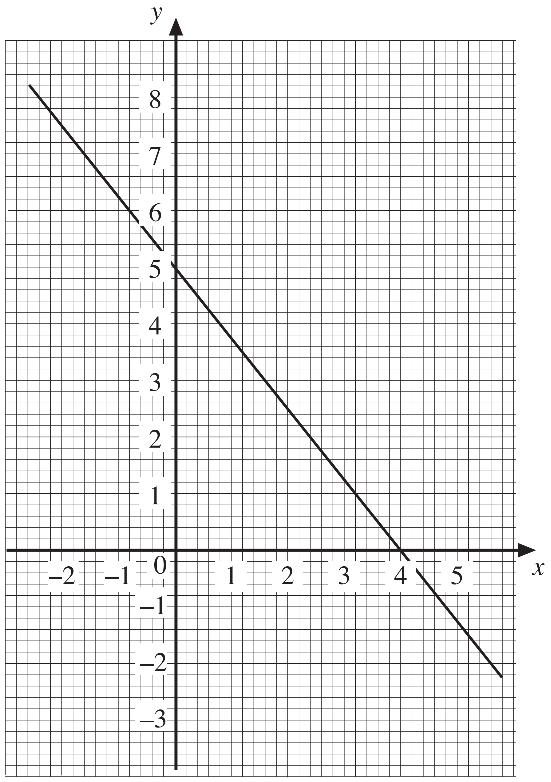
(ii) this cost.

(c) Copy and complete the following statement

"Method .... is always cheaper for customers who use more than .... units."

1

9. (a)



(i) The graph of $5x + 4y = 20$ is shown on the diagram above.

On a copy of the diagram, draw the graph of $y = 2x$.

(ii) Use the graphs to find the solution of the simultaneous equations

$$5x + 4y = 20$$
$$y = 2x$$

Give the value of x and the value of y to one decimal place.

(b) Calculate the **exact** solution of the simultaneous equations

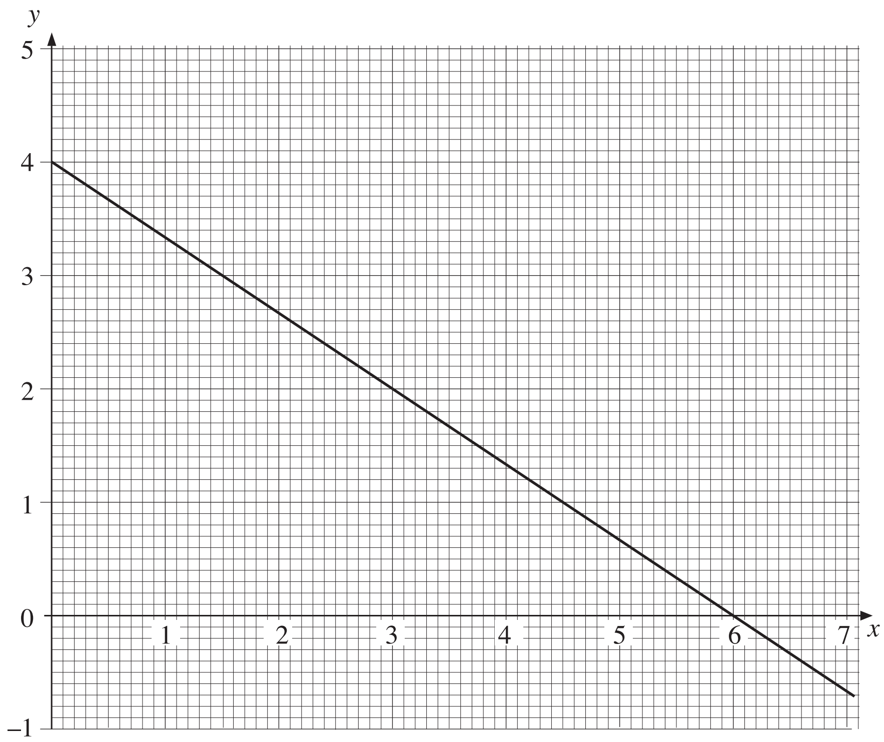
$$5x + 4y = 20$$
$$2x - y = 0$$

10. The line with equation $2x + 3y = 12$ is drawn on the following grid.

Solve the simultaneous equations

$$y = 2x - 1$$
$$2x + 3y = 12$$

by a graphical method.

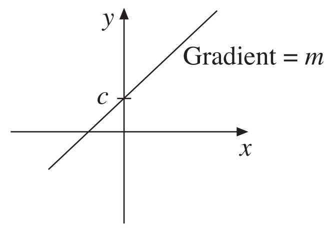


2 Graphs of Common Functions

Linear Functions

Linear functions are always straight lines and have equations which can be put in the form

$$y = mx + c$$

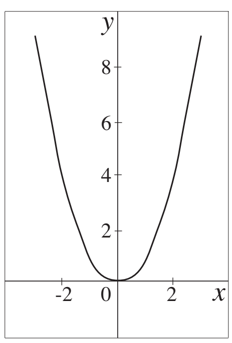


Quadratic Functions

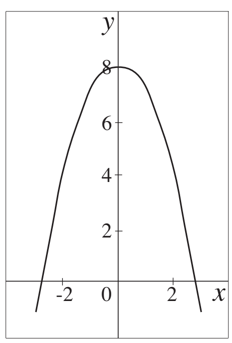
Quadratic functions contain an x^2 term as well as multiples of x and a constant. Some examples are:

$$y = 2x^2 \qquad y = x^2 - x + 5 \qquad y = 6 - x^2$$

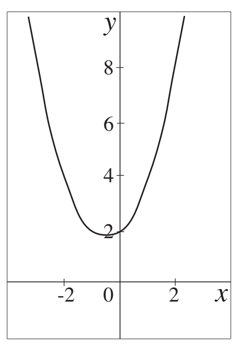
The following graphs show 3 examples.



$y = x^2$



$y = 8 - x^2$



$y = x^2 + x + 2$

Note that each curve has either a maximum or a minimum point which lies on its axis of symmetry. The curve has a maximum point when the coefficient of x^2 is negative as in the second example, or minimum if the coefficient of x^2 is positive. Also the curve can intersect the x-axis twice, just touch it once or never meet the x-axis.

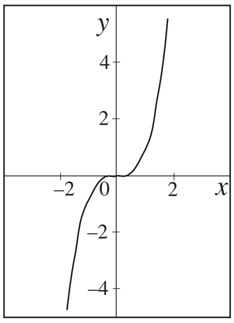
Cubic Functions

Cubic functions involve an x^3 term and possibly x^2, x and constant terms as well. Some examples are:

$$y = x^3, \qquad y = x^3 + 3x^2 + 4x - 8, \qquad y = x^3 - 5, \qquad y = x^3 - x + 1$$

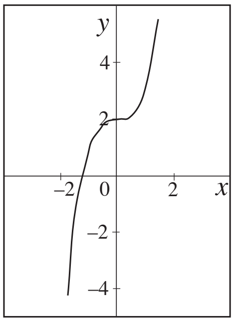
The graphs below show some examples.

(a)



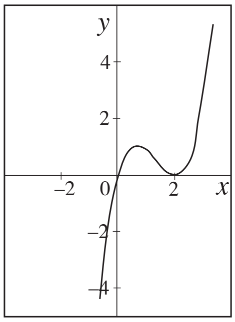
$y = x^3$

(b)



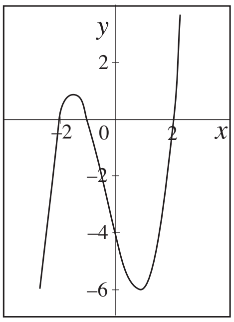
$y = x^3 + 2$

(c)



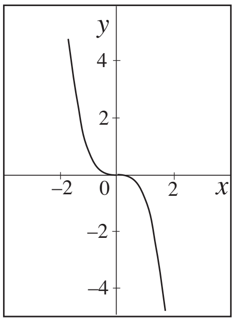
$y = x^3 - 4x^2 + 4x$

(d)



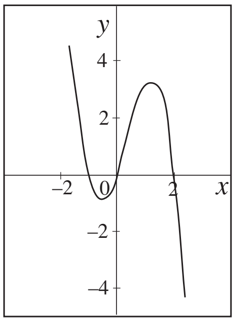
$y = x^3 + x^2 - 4x - 4$

(e)



$y = -x^3$

(f)



$y = -x^3 + x^2 + 2x$

The graph of a cubic function can intersect the x-axis once as in examples (a), (b) and (e), touch the axis once and intersect it once as in example (c) or intersect the x-axis three times as in examples (d) and (f).

In examples (c), (d) and (f) the curve has a local minimum and a local maximum.

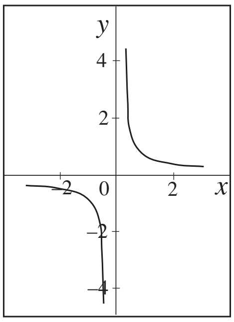
Note how the shape of the curve changes when a $-x^3$ is introduced. Compare examples (a) and (e).

Reciprocal Functions

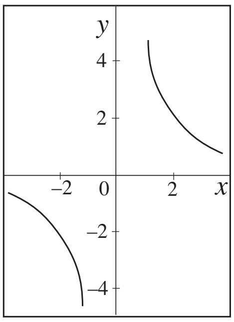
Reciprocal functions have the form of a fraction with x as the denominator. Examples of reciprocal functions are:

$$y = \frac{1}{x} \qquad y = \frac{10}{x}, \qquad y = \frac{-2}{x}, \qquad y = \frac{1}{5x}$$

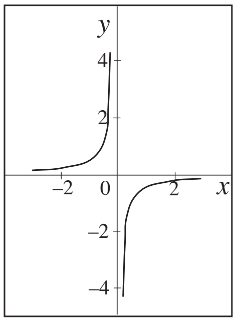
The graphs below show some examples.



$y = \frac{1}{x}$



$y = \frac{4}{x}$



$y = -\frac{1}{2x}$

The curves are split into two distinct parts. The curves get closer and closer to the axes as is clear in the diagrams. The curves have two lines of symmetry, $y = x$ and $y = -x$.

Exercises

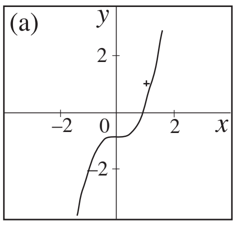
1. State whether each equation below would produce the graph of a *linear, quadratic, cubic* or *reciprocal* function.

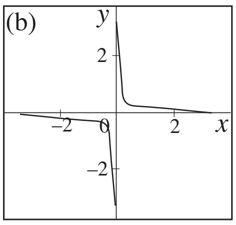
   (a) $y = \frac{4}{x}$ (b) $y = x^2 - 2$

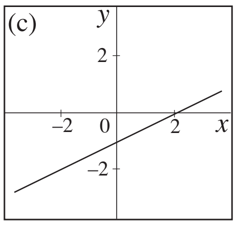
   (c) $y = x - 2$ (d) $y = x^3 + x^2 + 4$

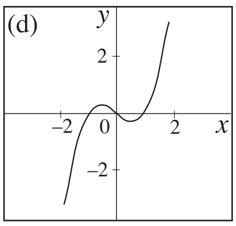
   (e) $y = x^2 + x$ (f) $y = \frac{-1}{7x}$

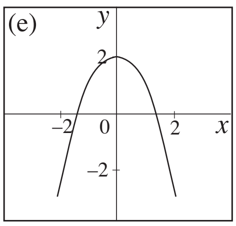
2. Each of the following graphs is produced by a *linear, quadratic, cubic* or *reciprocal* function. State which it is for each graph.

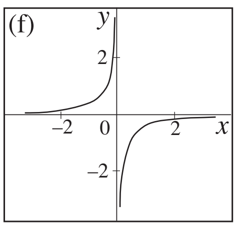




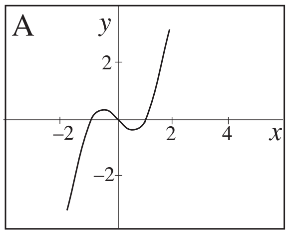


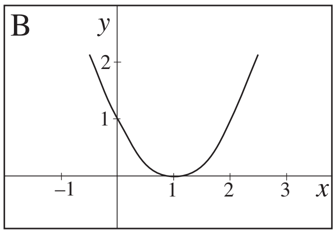


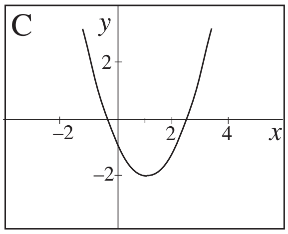


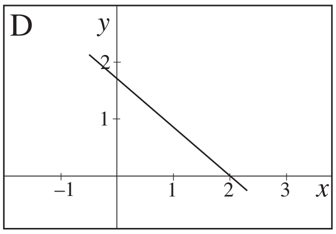


3. One of the graphs shown below is $y = x^2 - 2x + 1$. Which one?

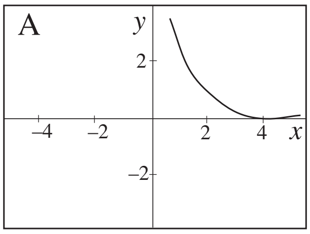


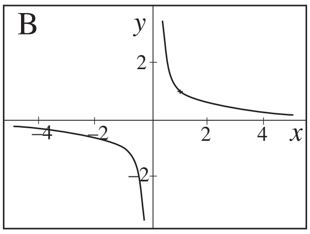


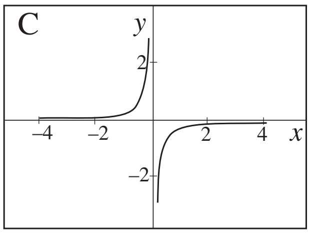


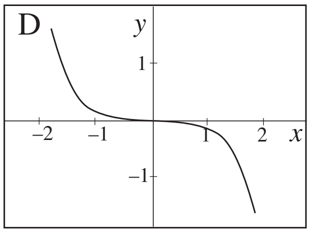


4. Which of the graphs shown below are reciprocal functions?



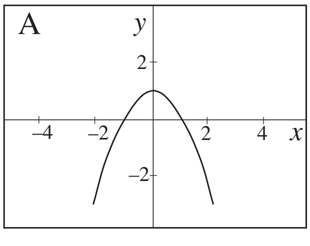


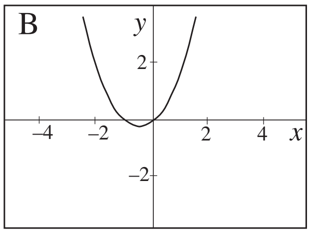


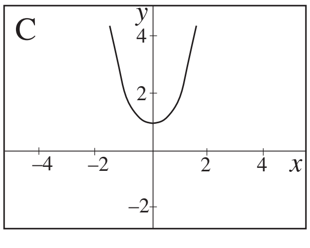


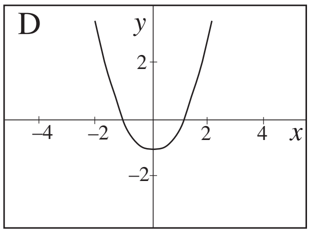
5. Each equation below has been plotted. Select the correct graph for each equation.

(a) $y = x^2 + 1$ (b) $y = x^2 - 1$ (c) $y = 1 - x^2$ (d) $y = x^2 + x$





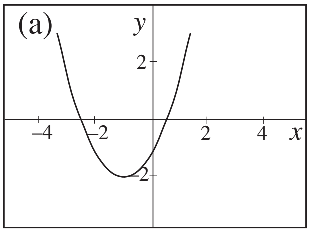


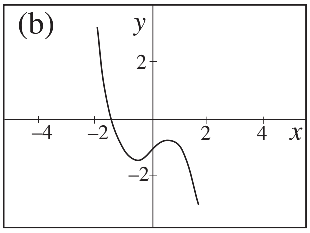


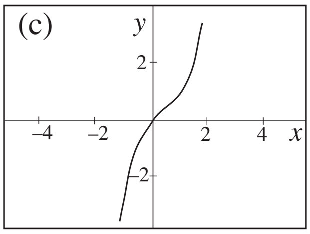
6. Match each graph below to the appropriate equation.

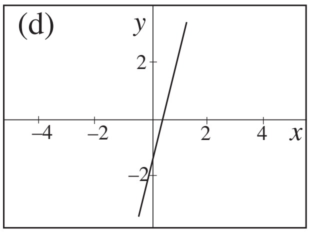
A $y = -x^3 + x - 1$ B $y = x^3 - x^2 + x$

C $y = 4x - 1$ D $y = x^2 + 2x - 1$







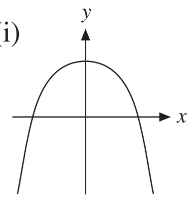


7. (a) Which of the following equations are illustrated by the graphs shown?

Write the equation illustrated beside the number of each graph.

$y = -x$ $y = 2 - x$ $y = 1 - x^2$

$y = x^2$ $2y = 2 + x$ $xy = 1$

(i)


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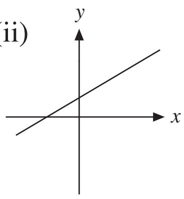

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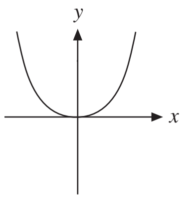
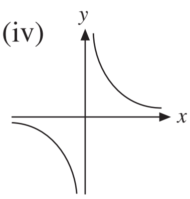
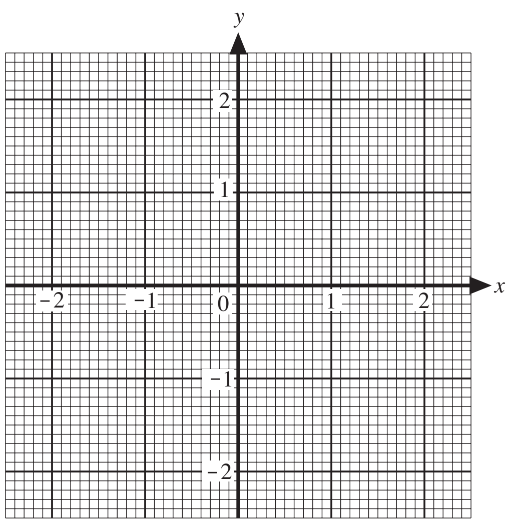


(iv)


(b) Sketch a graph of the equation $y = x^3 + 1$ on a copy of this graph



3 Graphical Solutions of Equations

Equations of the form $f(x) = g(x)$ can be solved graphically by plotting the graphs of $y = f(x)$ and $y = g(x)$. The solution is then given by the *x*-coordinate of the point where they intersect.

Worked Example 1

Find any positive solutions of the equation

$$x^2 = \frac{1}{x} + x$$

by a graphical method.

Solution

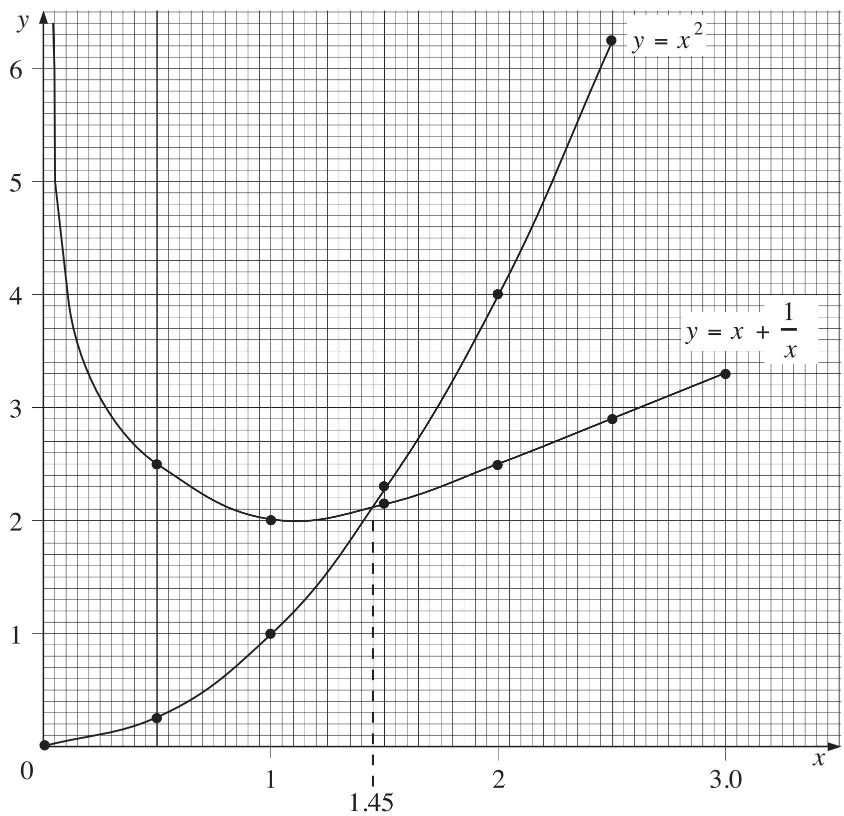
Completing the table below provides the points needed to draw the graphs $y = x^2$ and $y = \frac{1}{x} + x$.

x	0	0.5	1.0	1.5	2.0	2.5	3.0
x^2	0	0.25	1	2.25	4	6.25	9
$\frac{1}{x}+x$	Infinity	2.5	2	2.17	2.5	2.9	3.33

Where necessary the values have been rounded to 2 decimal places.

The graph below shows $y = x^2$ and $y = \frac{1}{x} + x$.

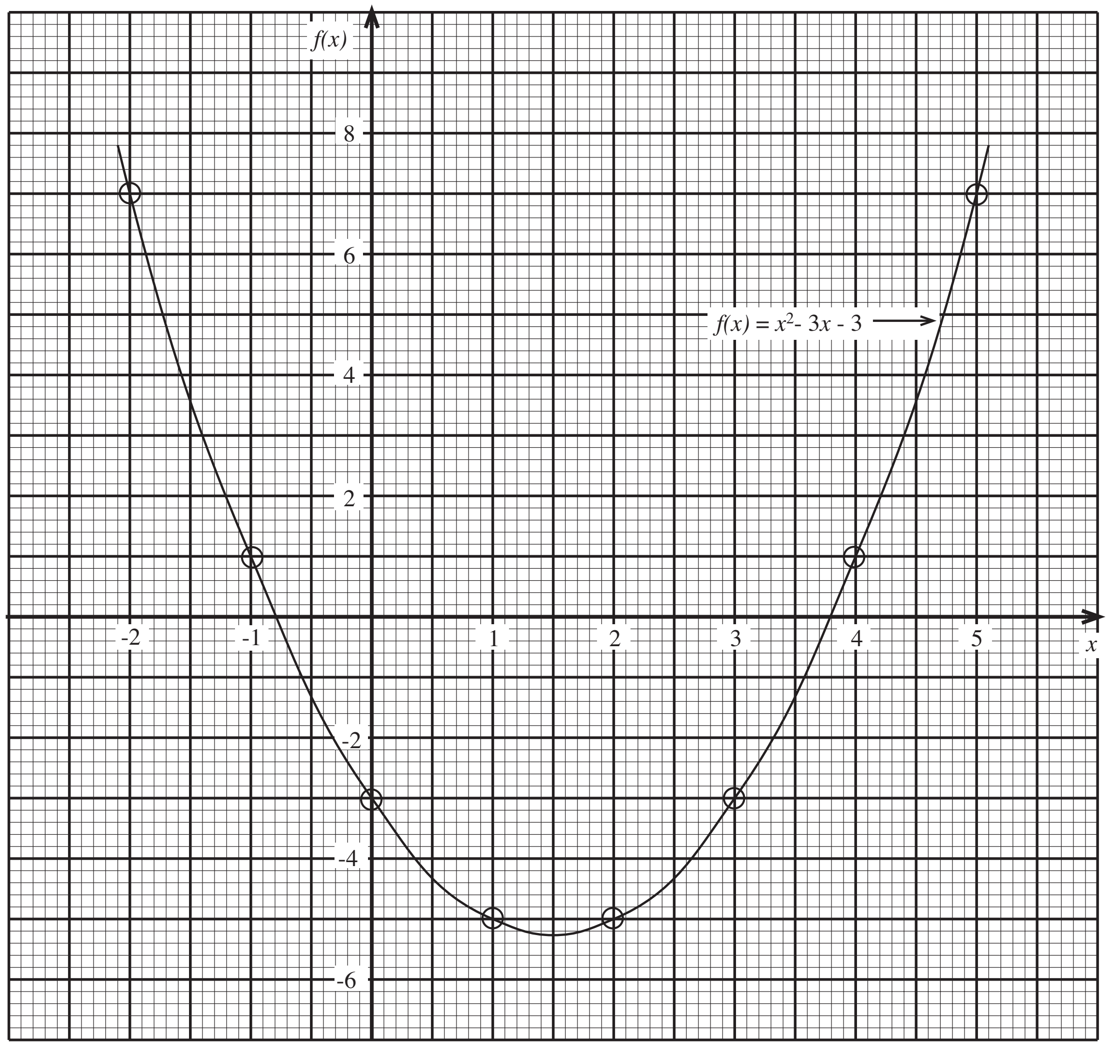
The curves intersect where $x = 1.45$ and so this is the solution of the equation.



Worked Example 2

The graph below represents the function

$$f(x) = x^2 - 3x - 3$$



Use the graph to determine

(a) the value of $f(x)$ when $x = 2$

(b) the value of $f(x)$ when $x = -1.5$

(c) the value of x for which $f(x) = 0$

(d) the minimum value of $f(x)$

(e) the value of x at which $f(x)$ is a minimum

(f) the solution of $x^2 - 3x - 3 = 5$

(g) the interval on the domain for which $f(x)$ is less than -3.

Solution

Using the graph:

(a) $f(2) = -5$

(b) $f(-1.5) \approx 3.6$

(c) Intercepts with x-axis are $x = -0.8, 3.8$

(d) $f\,\text{min} = -5.3$

(e) $x = 1.5$

(f) $x = -1.7$ and 4.7

(g) $0 < x < 3$

Note

The domain of a function is the values of x for which the function is defined. This is covered in Unit G4.

Worked Example 3

Given that $y = 2x^2 - 9x + 4$

(a) copy and complete the table below

x	–2	–1	0	2	4	6
y	30		4		0	22

(b) using a scale of 1 cm to represent 1 unit on the x-axis and 2 cm to represent 5 units on the y-axis, draw the graph of $y = 2x^2 - 9x + 4$ for $-2 \le x \le 6$

(c) **use your graph** to solve the equation

$$2x^2 - 9x + 4 = 15$$

Solution

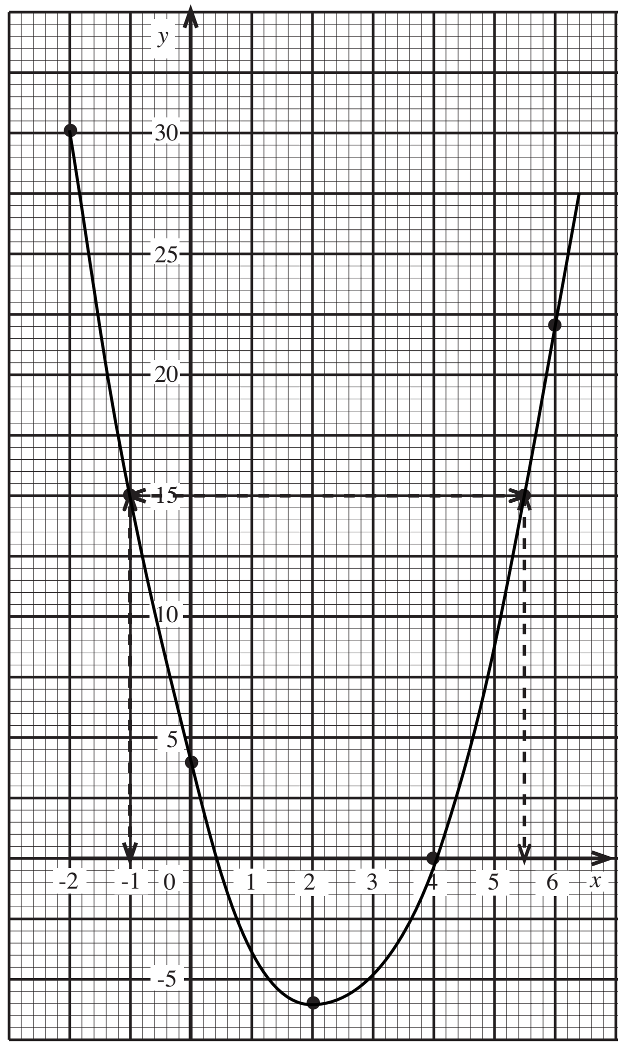
(a) Missing values:

$$f(-1) = 2 \times (-1)^2 - 9 \times (-1) + 4 = 15$$

$$f(2) = 2 \times (2)^2 - 9 \times 2 + 4 = -6$$

x	–2	–1	0	2	4	6
y	30	15	4	–6	0	22

(b)



(c) Using the intersection of $y = 15$ with $y = 2x^2 - 9x + 4$ gives estimates of the solution of

$$2x^2 - 9x + 4 = 15$$

as $x = -1$ and $x = 5.5$ (see graph above).

Worked Example 4

(a) The grid on the following page shows the line, l, which passes through the points $Q\ (0, -1)$ and $R\ (3, 2)$.

(i) Determine the gradient of the line, l.

(ii) Write down the equation of the line, l.

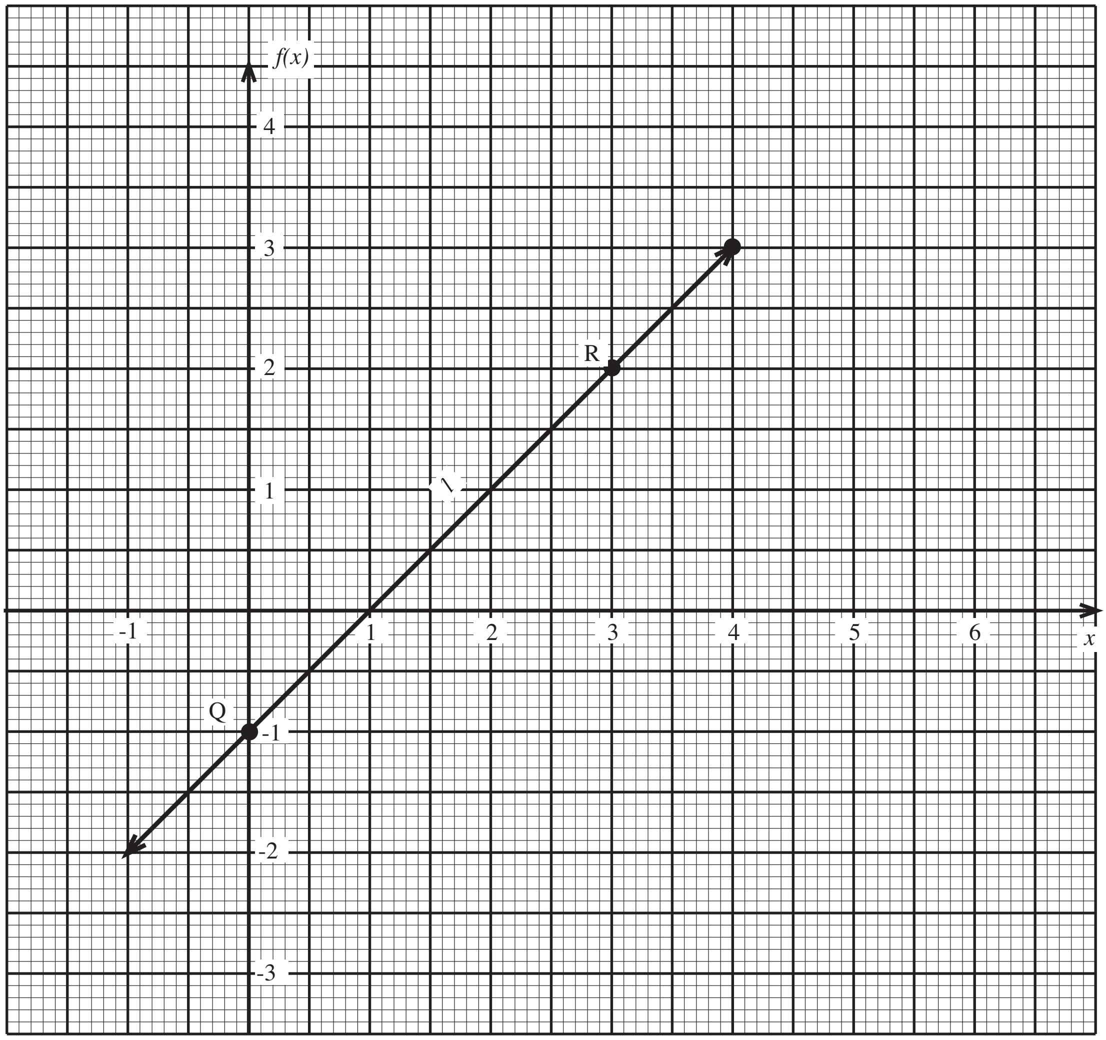
(b) The table below shows three of the values of $f(x) = x^2 - 4x + 3$ for values of x from 0 to 4.

x	0	1	2	3	4
y	3		-1	0	

(i) Copy the table and insert the missing values of $f(x)$.

(ii) On a copy of the grid below, draw the graph of $f(x) = x^2 - 4x + 3$.

(iii) Using the graphs, write down the coordinates of the points of intersection of the line, *l*, and the graph of $f(x)$.



Solution

(a) (i) Gradient $= \dfrac{2 - (-1)}{3 - 0} = \dfrac{3}{3} = 1$

(ii) y-intercept is -1, so equation of l is

$$y = -1 + x$$

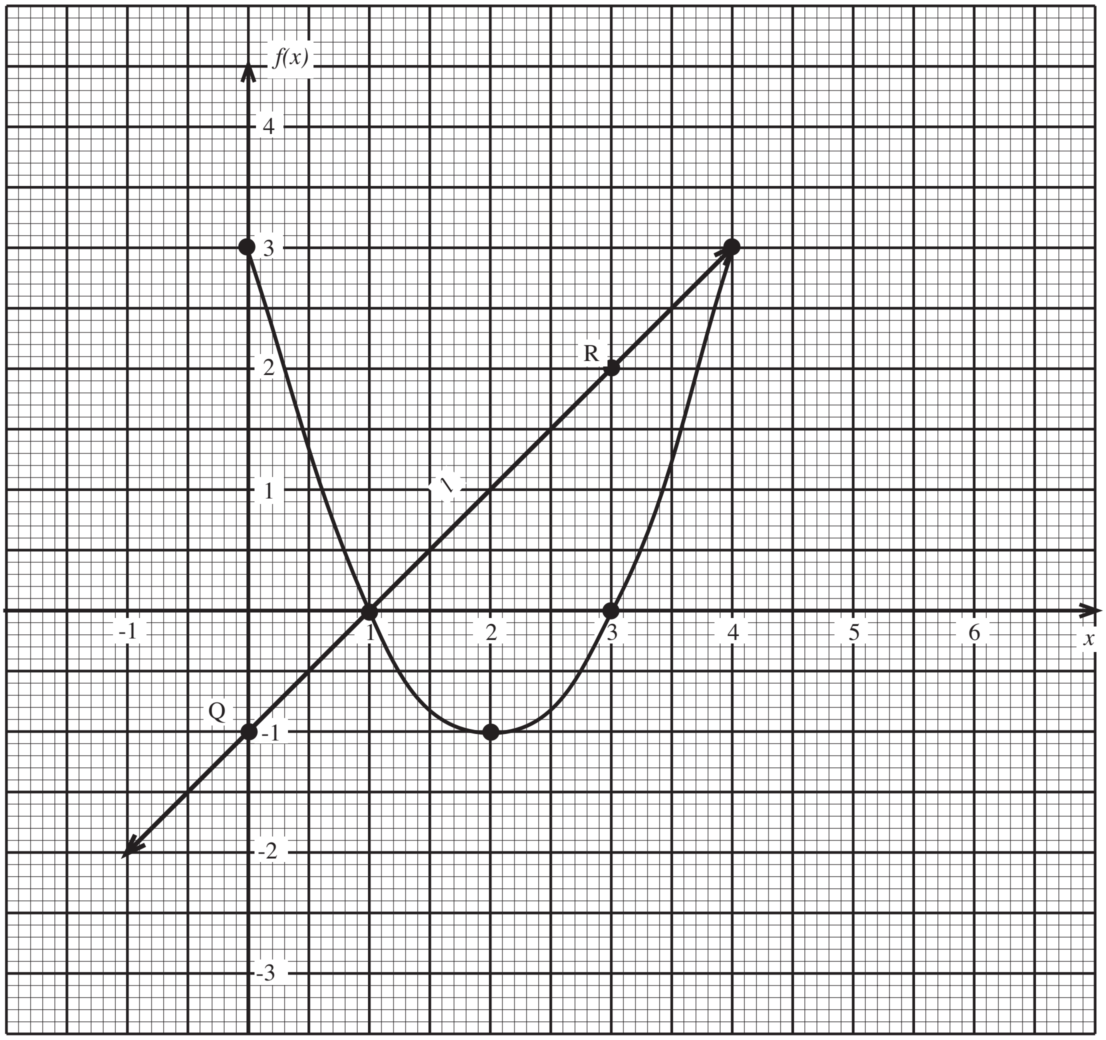
(Alternative method: equation is of the form $y = mx + c = x + c$;
to pass through the point $(0, -1)$, $-1 = 0 + c \Rightarrow c = -1$

$\Rightarrow y = x - 1$)

(b) (i)

x	0	1	2	3	4
$f(x)$	3	0	–1	0	3

(ii)



(iii) Points of intersection at $(1, 0)$ and $(4, 3)$.

Exercises

1. Draw the graph of $y = 3^x$ for $0 \le x \le 2$. Use the graph to solve the equations $4 = 3^x$ and $5 = 3^x$.

2. Solve the quadratic equation $x^2 - x - 2 = 0$ by plotting the graphs $y = x^2$ and $y = x + 2$.

3. Find the x-coordinates of the two points where the lines $y = x^2 - 2$ and $y = x + 4$ intersect. Write down the quadratic equation which has the two solutions you found from the points of intersection.

4. Find the solutions of the following equations

(a) $2x - x^2 = x^3$ (b) $3^x = x - 2$

(c) $x^4 = 8 - x^2$ (d) $\dfrac{1}{x^2} = \dfrac{x^3}{8} - 1$

5. Describe 3 different ways to find solutions of the equation

$$x^3 - 8x + 5 - \frac{3}{x} = 0$$

6. Draw the graph $y = x^2 + 2x$ for $-3 \le x \le 2$. Use the graph to solve the following equation,

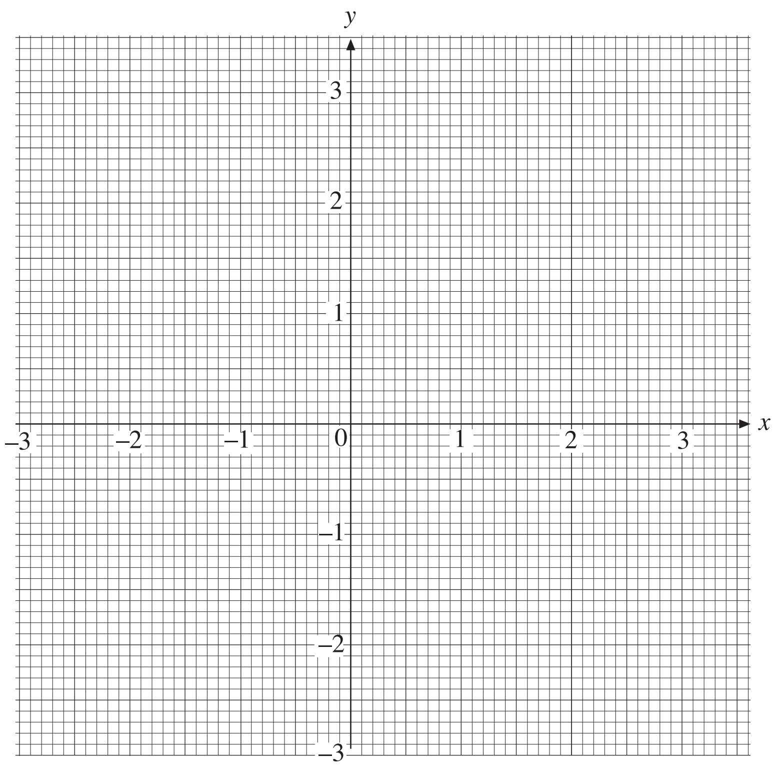
$$x^2 + 2x - a = 0$$

if (a) $a = 3$ (b) $a = 2$ (c) $a = -\dfrac{1}{2}$

For what value of a is there only one solution?

For what value of a are there no real solutions to the equation?

7.

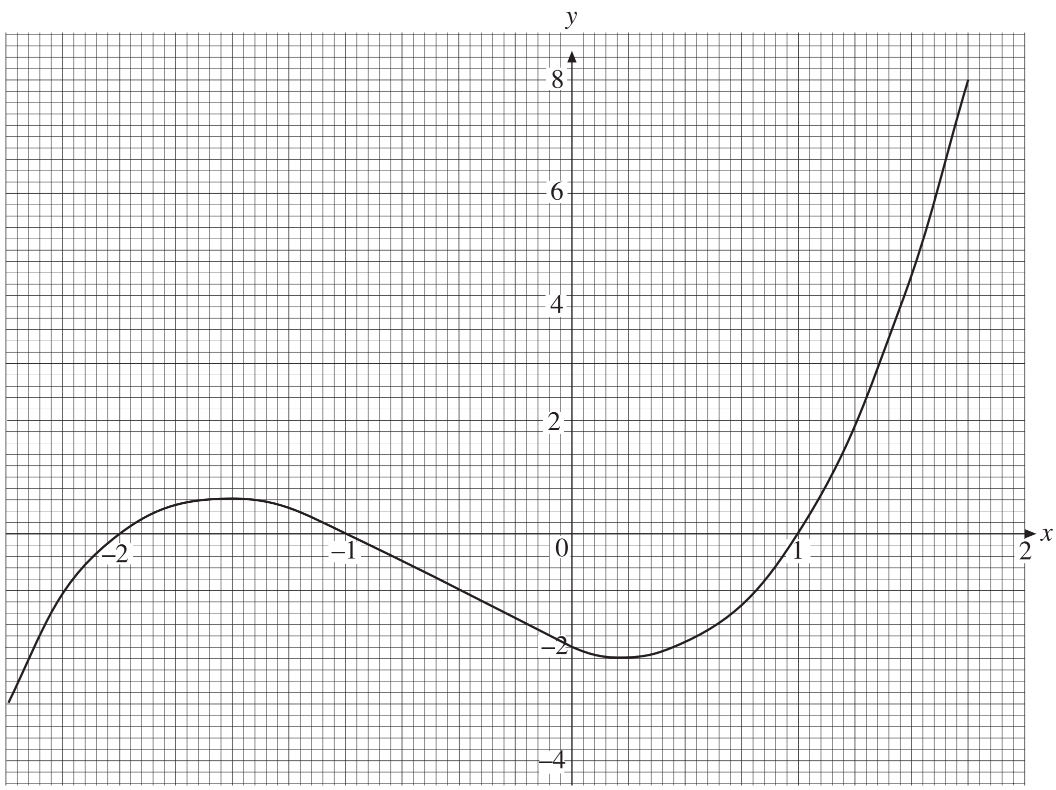


Use a graphical method to solve the equation

$$\frac{1}{x} = x^2 - 1$$

You **must** show all your working.

8.



The diagram above shows the graph of $y = x^3 + 2x^2 - x - 2$.

(a) Use the graph to find the solutions of the equation

$$x^3 + 2x^2 - x - 2 = 0$$

(b) By drawing the graph of $y = 2x^2$ on a copy of the diagram, find the solution of the equation

$$x^3 - x - 2 = 0$$

(c) Use the graph to find solutions of the equation

$$x^3 + 2x^2 - x - 1 = 0$$

9.

x	–2	–1	0	1	2	3
y	7	0			3	12

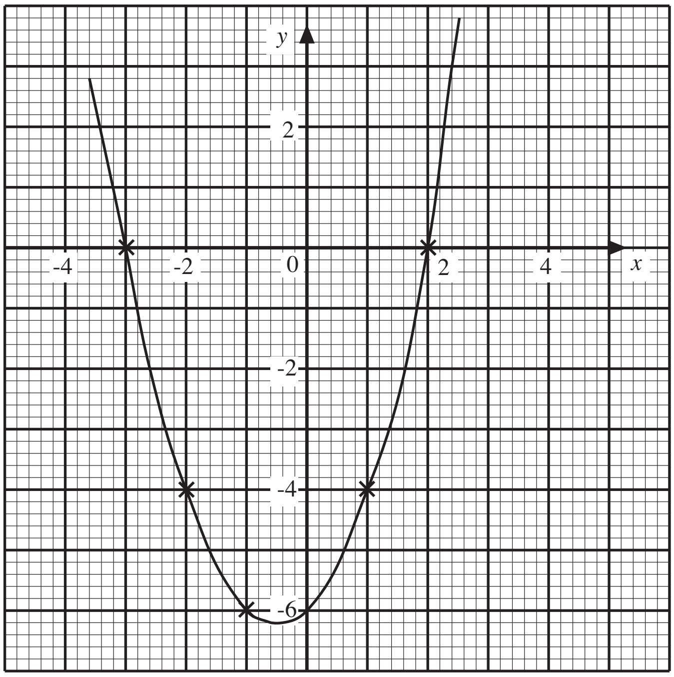
The table above shows some values of $y = 2x^2 - x - 3$ for values of x from -2 to 3.

(a) What are the missing values of y?

(b) On a graph, plot the points recorded in your completed table at (a) above, and draw a smooth curve through the points.

(c) Use your graph to find the values of x for which $2x^2 - x - 3 = 0$.

10.



The diagram above shows the graph of the function $y = px^2 + qx + r$.

(a) Determine the values of *p, q* and *r*.

(b) State TWO ways in which the graphs of the functions $y = px^2$ and $y = px^2 + qx + r$ are similar.

(c) State ONE way in which the graphs of the two functions is different.

11. (a) Given that $f(x) = x^2 + x - 2$, copy and complete the table below.

x	-3	-2	-1	0	1	2
$f(x)$	4		-2		0	

(b) Using 2 cm to represent 1 unit on both axes, draw the graph of

$$f(x) = x^2 + x - 2 \text{ for } -3 \le x \le 2.$$

(c) On the graph of $f(x) = x^2 + x - 2$, draw the graph of $g(x) = x - 1$ using the values from the table shown below.

x	-2	3
$g(x)$	-3	2

(d) Using the graphs, write down the coordinates of the points where the two graphs intersect.

12. The diagram below shows the graph of the function $f(x) = x^2 - 2x - 3$ for $a \le x \le b$. The tangent to the graph at (2, –3) is also drawn.

Use the graph to determine the

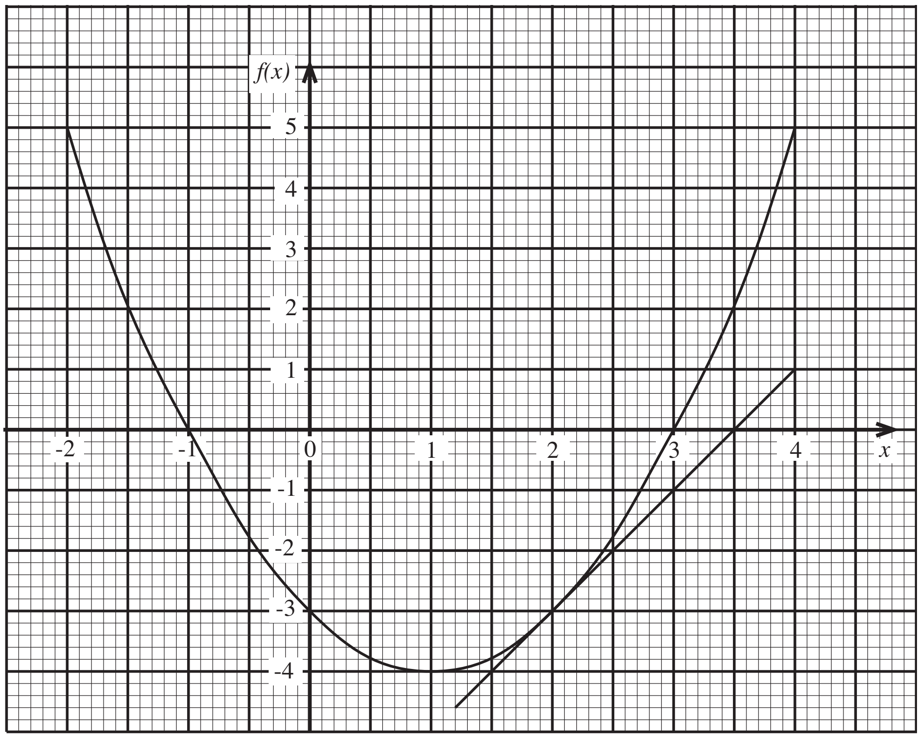
(a) values of a and b which define the domain of the graph.

(b) values of x for which $x^2 - 2x - 3 = 0$

(c) coordinates of the minimum point on the graph

(d) whole number values of x for which $x^2 - 2x - 3 < 1$

(e) gradient of $f(x) = x^2 - 2x - 3$ at $x = 2$.



13. (a) Given that $y = \frac{1}{2}x^3$, copy and complete the table below.

x	−2	−1	0	1	2	3
y		−0.5	0		4	13.5

(b) **Using scales of 2 cm to represent 1 unit on the *x*-axis, and 1 cm to represent 1 unit on the y-axis**, draw the graph of the function y for $-2 \le x \le 3$

(c) Using the graph

(i) solve the equation, $\frac{1}{2}x^3 = 4$

(ii) determine the values of x for which $\frac{1}{2}x^3 \le 4$.

(d) Using the same axes and scales,

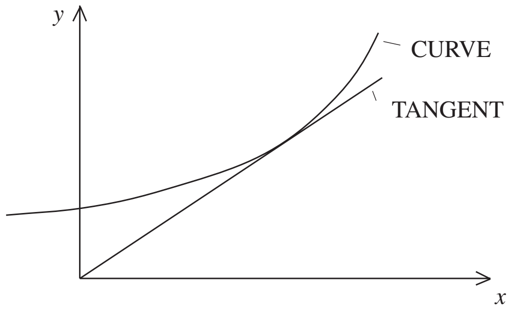
(i) draw the graph of $y = 2$

(ii) write down the equation in x whose root is given by the intersection of the graphs, $y = 2$ and $y = \frac{1}{2}x^3$.

(CXC)

4 Tangents to Curves

A tangent is a line that touches a curve at one point only, as shown opposite.



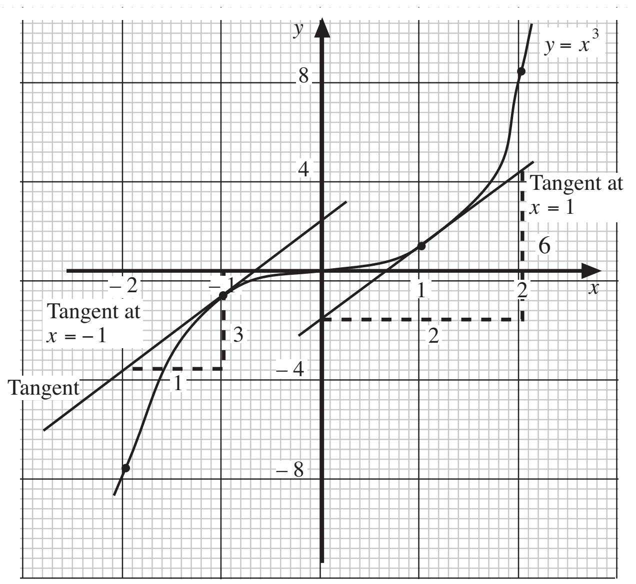
The gradient of the tangent gives the *gradient of the curve* at that point. The gradient of the curve gives the *rate* at which a quantity is changing. For example, the gradient of a distance-time curve gives the rate of change of distance with respect to time, which gives the velocity.

Worked Example 1

Draw the graph of $y = x^3$ for $-2 \le x \le 2$. Draw tangents to the curve at $x = -1$ and $x = 1$. Find the gradients of these tangents.

Solution

The graph of $y = x^3$ is shown below. The tangents have been drawn at $x = -1$ and $x = 1$.



Using the triangles shown under each tangent, show that the gradients of both tangents are 3.

Worked Example 2

The height, h, of a ball thrown straight up in the air varies so that at time, t, $h = 8t - 5t^2$.

Plot a graph of h against t and use it to find:

(a) the speed of the ball when $t = 0.6$,

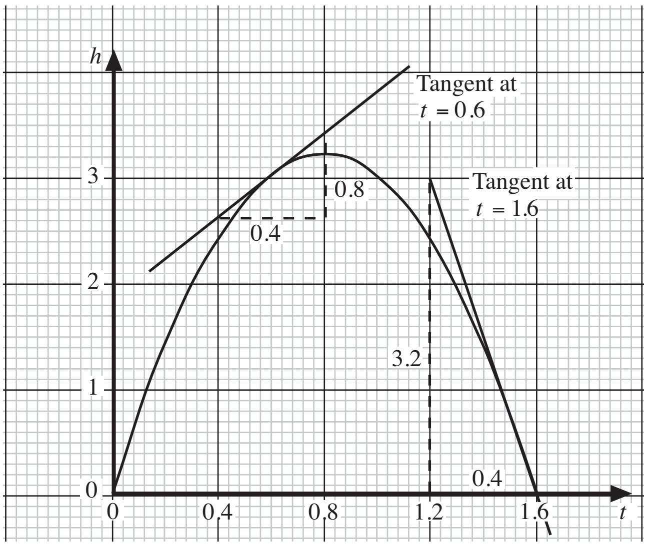
(b) the greatest speed of the ball.

Solution

The table below gives the values needed to plot the graph.

t (s)	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6
h (m)	0	1.4	2.4	3.0	3.2	3.0	2.4	1.4	0

The graph is shown below.



(a) A tangent has been drawn at the point where $t = 0.6$. The gradient of this tangent is $\frac{0.8}{0.4} = 2$. So the speed of the ball is 2 m/s.

(b) The speed of the ball is a maximum when the curve is steepest, that is at $t = 0$ and $t = 1.6$. At $t = 1.6$ the gradient is $\frac{-3.2}{0.4} = -8$. So the speed is 8 m/s.

The '–' sign indicates that the ball is moving down rather than up. You can say the ball moves down with speed 8 m/s or that the velocity of the ball is –8 m/s.

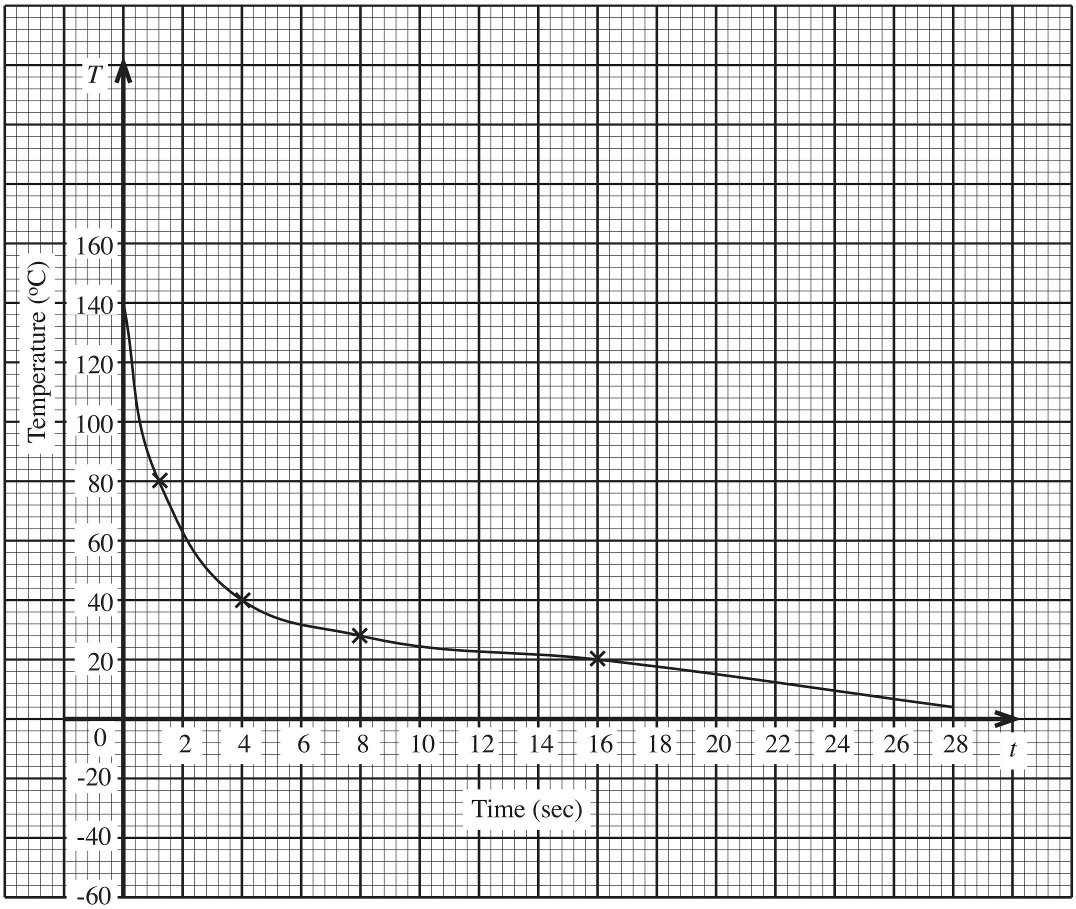
Worked Example 3

The following graph represents the cooling curve for a certain liquid.

Use the graph to estimate

(a) the temperature when the time, t, is 12 secs.

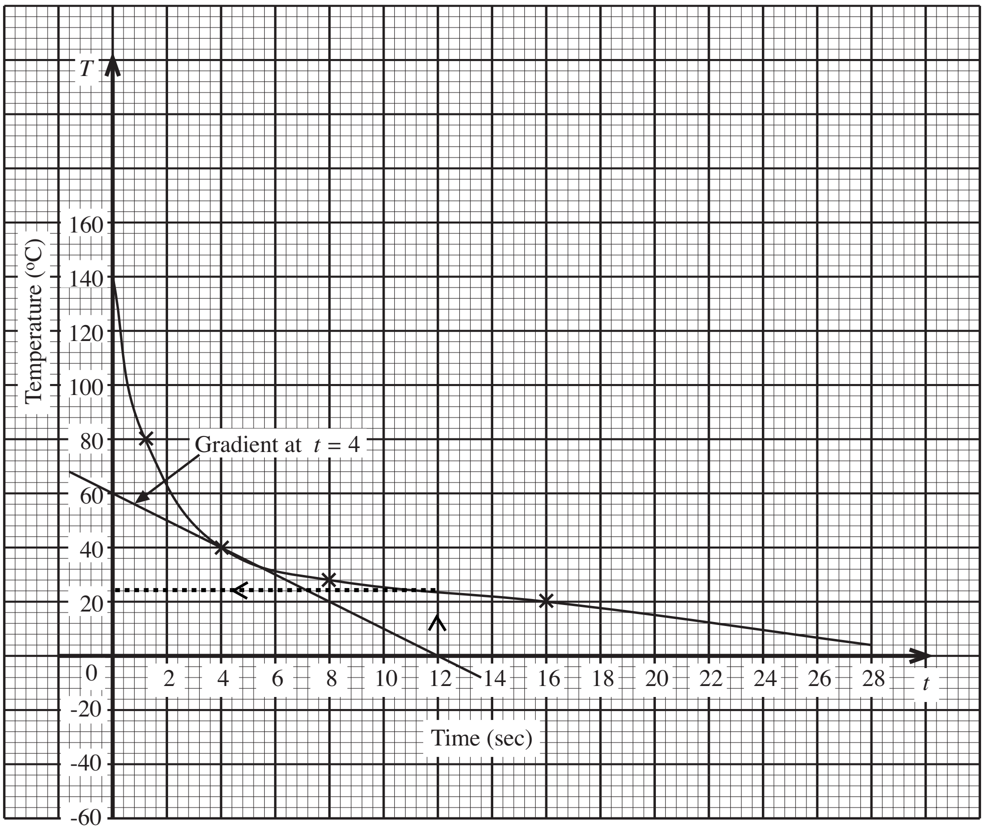
(b) the gradient of the curve when the time, t, is 4 secs.



Solution

(a) From the graph, temperature $\approx 24\,°C$ at time 12 secs.

(b) From the graph (see next page), gradient $\approx \dfrac{0-60}{12-0} = 5°C$ per sec.

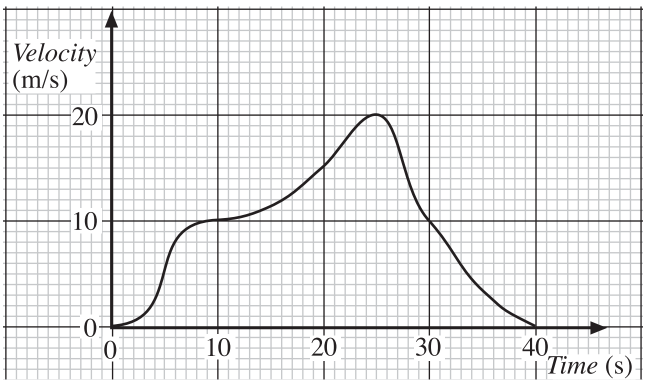


Worked Example 4

The graph shows how the velocity of a car changes. Find:

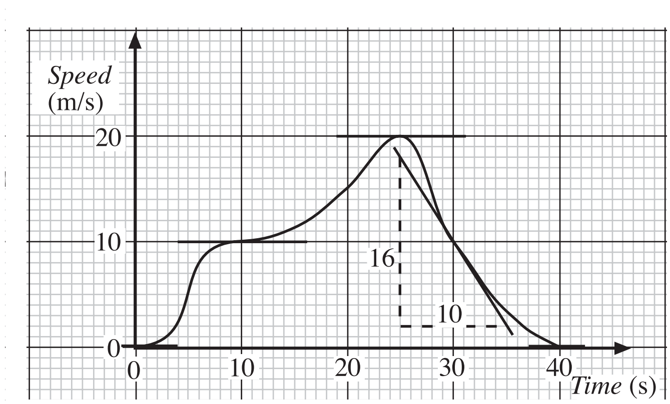
(a) the time when the acceleration of the car is zero,

(b) the acceleration when $t = 30$.



Solution

The acceleration of the car is given by the gradient of the velocity-time graph. There are 4 points where the gradient is zero, at $t = 0$, $t = 10$, $t = 25$ and $t = 40$. At each of these points a horizontal tangent can be drawn to the curve as shown opposite.



A tangent has been drawn to the curve at $t = 30$.

The gradient of this curve is $\frac{-16}{10} = -1.6$, so the acceleration is -1.6 m/s^2.

Exercises

1. (a) Draw the graph of $y = x^2$ for $0 \le x \le 4$.

   (b) By drawing tangents find the gradient of the curve at $x = 0$, $x = 1$, $x = 2$, $x = 3$ and $x = 4$.

   (c) Comment on any patterns that are present in your answers.

2. The height, h, of a ball at time, t, is given by, $h = 10t - 5t^2$.

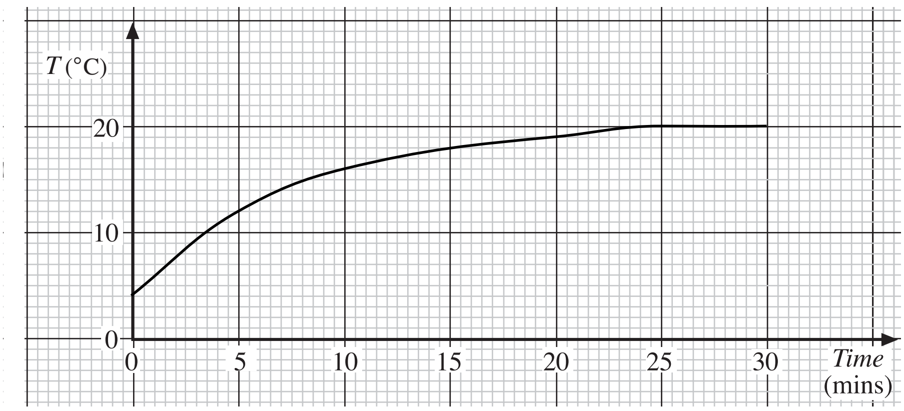
   The ball travels straight up and down.

   (a) Draw a graph of h against t for $0 \le t \le 2$.

   (b) Use the graph to find the velocity of the ball when $t = 0.5$ and $t = 1.2$.

   (c) Find the maximum speed of the ball.

3. The graph below shows how the temperature of a can of drink increases after it has been taken out of a fridge.



   Find the rate of change of temperature with respect to time, when;

   (a) $t = 0$,

   (b) $t = 10$,

   (c) $t = 15$.

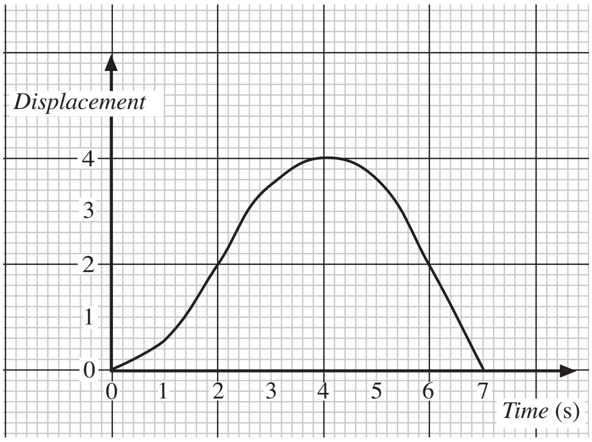
4. A car moves so that its velocity, v, and time, t, is given by $v = t^2 - 8t + 16$.

   (a) Plot a graph of velocity against time for $0 \le t \le 4$.

   (b) Find the gradient of the curve when $t = 0, 1, 2, 3$ and 4.

   (c) Use your results to (b) to sketch a graph of acceleration against time for the graph.

5.



The graph shows how the displacement of an object varies with time.

(a) Copy the graph and by drawing tangents estimate the velocity of the object when $t = 0, 1, 2, 3, 4, 5, 6$ and 7.

(b) Use your results to part (a) to draw a velocity-time graph.

(c) Consider your velocity-time graph and sketch an acceleration-time graph.

6. Draw the graph of $y = x^3$ for $-3 \le x \le 3$.

(a) Find the gradient of the curve at $x = -3, -2, -1, 0, 1, 2$ and 3.

(b) Can you predict how to calculate the gradient of $y = x^3$ for any value of x?

7. Draw a graph of $y = \sin x$ for $0 \le x \le 360°$.

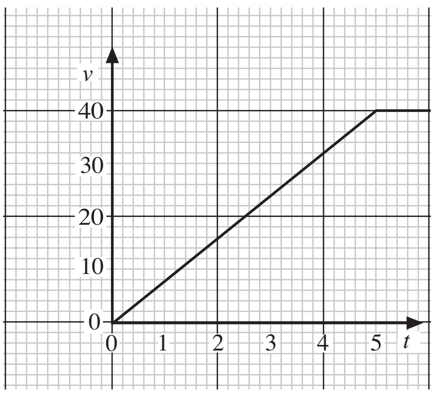
(a) For what values of x is the gradient of the curve zero?

(b) Find the maximum and minimum values of the gradient of the curve $y = \sin x$, and state the values of x for which they are obtained.

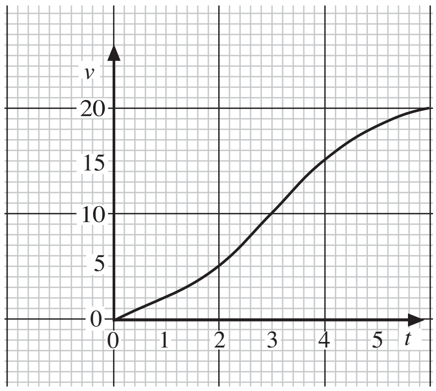
(c) Use these results to draw a graph of the gradient of $y = \sin x$ against x.

8. For each of the following velocity-time graphs, sketch an acceleration-time graph showing the maximum and minimum values of the acceleration.

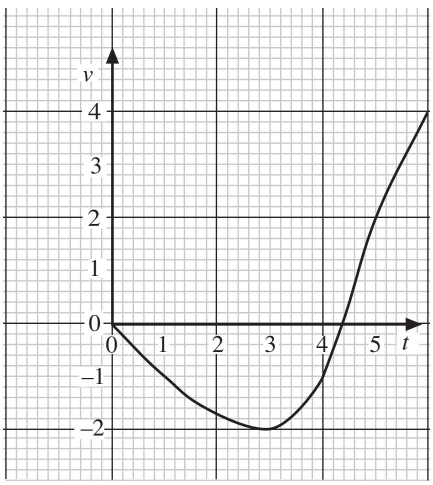
(a)



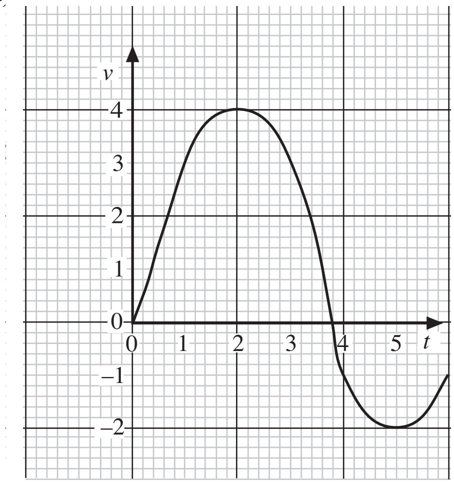
(b)



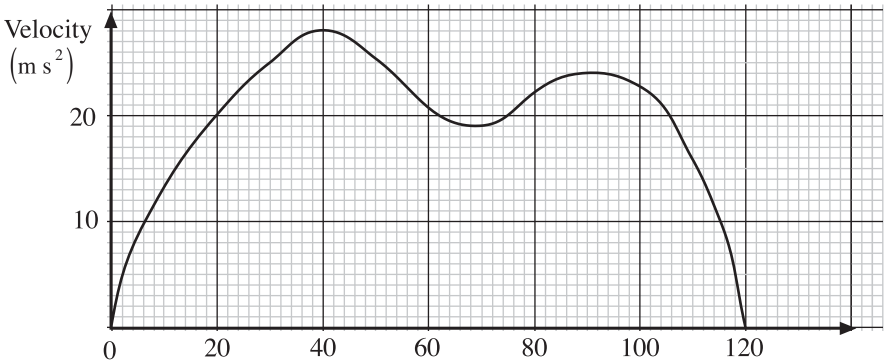
(c)



(d`



9. Here is a velocity-time graph of a car travelling between two sets of traffic lights.



Calculate an estimate for the acceleration of the car when the time is equal to 20 seconds.

10. The temperature, K, of a liquid t minutes after heating is given in the table below.

***t* (time in minutes)**	0	10	20	30	40	50	60
***K* (Temp. in °C)**	84	61	40	29	27	26	25

(a) (i) Using a scale of 2 cm to represent 10 minutes on the horizontal axis and a scale of 2 cm to represent 10 degrees on the vertical axis, construct a temperature-time graph to show how the liquid cools in the 60 minute interval.

Draw a smooth curve through all the plotted points.

(b) Use your graph to estimate

(i) the temperature of the liquid after 15 minutes

(ii) the rate of cooling of the liquid at $t = 30$ minutes.