

*Further Calculus***Answers****1 Rate of Change of the Gradient**

1. (a)  $6x$  (b)  $12x^2$  (c)  $2$  (d)  $0$  (e)  $\frac{2}{x^3}$   
 (f)  $24x - 24$  (g)  $0$  (h)  $e^x$  (i)  $-\frac{1}{x^2}$
2. (a)  $24x$  (b)  $-\frac{72}{x^5}$  (c)  $e^x$  (d)  $\frac{2}{x^3}$  (e)  $0$

**2 Stationary Points**

1. (a)  $x = -\frac{3}{4}$ , minimum (b)  $x = 2$ , minimum;  $x = -2$ , maximum  
 (c)  $x = \frac{-2+\sqrt{19}}{3}$ , minimum;  $x = \frac{-2-\sqrt{19}}{3}$ , maximum  
 (d)  $x = 1$ , point of inflection (e)  $x = \ln 4$ , minimum (f) none
2. (a) maximum at  $x = \frac{2+a}{3}$ ; minimum at  $x = a$   
 (b) minimum at  $x = \frac{2+a}{3}$ ; maximum at  $x = a$

**3 Differentiating Composite Functions**

1. (a)  $16(2x - 5)^7$  (b)  $10(x^2 + x^3)^9 (2x + 3x^2)$  (c)  $-\frac{1}{(x-2)^2}$   
 (d)  $-\frac{6}{(3x+1)^3}$  (e)  $\frac{1}{2\sqrt{x+1}}$  (f)  $6(e^x - x)^5(e^x - 1)$   
 (g)  $\frac{3}{(3x+4)}$  (h)  $\frac{1}{2\sqrt{x}} e^{\sqrt{x}}$
2. (a)  $x = 0$  (b)  $x = -4$  (c)  $x = 1$  (d)  $x = \pm 1$
3.  $-0.964^\circ\text{C}/\text{min}$ ;  $-0.290^\circ\text{C}/\text{min}$
4. 0.48 millions / year
5. (a)  $2e^{2x}$ ;  $\frac{e^{2x}}{2} + c$  (b)  $-e^{-x}$ ;  $-e^{-x} + c$
6.  $2xe^{x^2}$ ;  $\frac{1}{2}e^{x^2} + c$
7.  $\ln(x + 2) + c$

**4 Integration Again**

1. (a)  $\frac{(2x+1)^5}{10} + c$       (b)  $-\frac{1}{(x-5)} + c$   
 (c)  $2\sqrt{x+1} + c$       (d)  $\frac{1}{6}(4x-1)^{\frac{3}{2}} + c$
2. (a)  $\ln 2$       (b)  $\frac{1}{2} \ln 3$
3. (a)  $\frac{1}{3} \ln(3x+2) + c$       (b)  $\frac{1}{2} \ln(x^2+1) + c$       (c)  $\ln(1+e^x) + c$
4.  $\frac{1}{2}\left(1 - \frac{1}{e}\right)$

**5 Differentiating Products and Quotients**

1. (a)  $e^x(x+1)$       (b)  $x + 2x \ln x$       (c)  $\frac{-2(x^2+x+3)}{(x^2-3)^2}$       (d)  $\frac{xe^x}{(1+x)^2}$
2. (a)  $x = -1$       (b)  $x = 0, x = 0.607$       (d)  $x = 0$
3. Root of  $(1+x) \ln(1+x) = 1$
4.  $\frac{-3}{2(2x-1)^2} \sqrt{\frac{2x-1}{x+1}}$
5.  $\frac{e^x(2+x)}{2\sqrt{e^x(x+1)}}$
6.  $\frac{1}{\sqrt{x}} \left( \frac{1}{(x-2)} - \frac{\ln(x-2)}{2x} \right)$
7.  $x = 1$ , minimum ;  $x = -2$ , maximum
8.  $x = 0, 2$