Functions Essential information

The ***domain*** of a function is the value of x for which the function is defined.

For example, $f(x) = x^2 + 1$ for $x \geq 0$ (domain is $x \geq 0$)

$$g(x) = \frac{1}{x-1} + \frac{1}{x-2} \quad \text{for } 1 < x < 2 \quad (\text{domain is } 1 < x < 2)$$

The ***range*** of a function is the set of values that the function maps onto.

For example, if $f(x) = x^2,\ 0 \leq x \leq 5$,

the range of f is $0 \leq f(x) \leq 25$.

1 : 1 mapping is a function for which every value of $f(x)$ is unique; that is, if $f(a) \neq f(b)$ unless $a = b$.

For example, $f(x) = x + 1$ is a 1 : 1 mapping, but

$f(x) = x^2$ is *not* a 1 : 1 mapping as, for example,

$f(2) = 4 = f(-2)$

The ***composite function*** fg or $fg(x)$ is defined as $f(g(x))$

For example, if $f(x) = x + 1$ and $g(x) = x^2$, then

$$fg(x) = f(x^2) = x^2 + 1$$

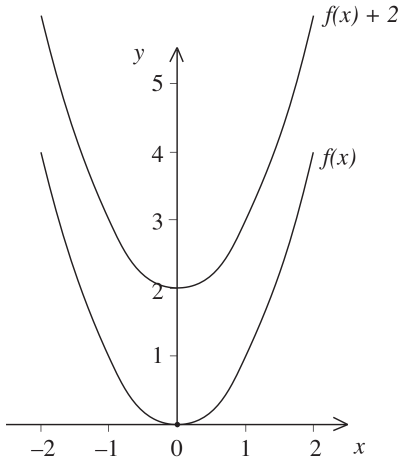
and

$$gf(x) = g(x+1) = (x+1)^2$$

The ***inverse function*** of f is denoted by f^{-1} and is such that

$$ff^{-1}(x) = x = f^{-1}f(x)$$

The inverse only exists if the function is a 1 : 1 mapping.



The graph of $y = f(x)$ is mapped onto the graph of $y = f(x) + 2$ by translating it up 2 units.

> In general $f(x) + a$ moves a curve up a units and $f(x) - a$ moves it down a units, where a is a positive number.

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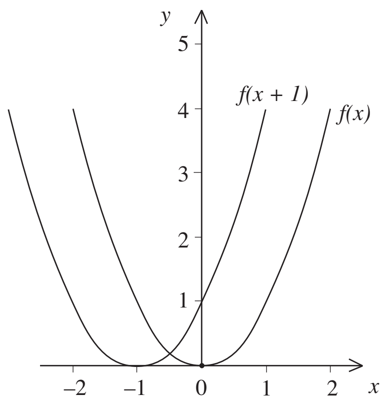
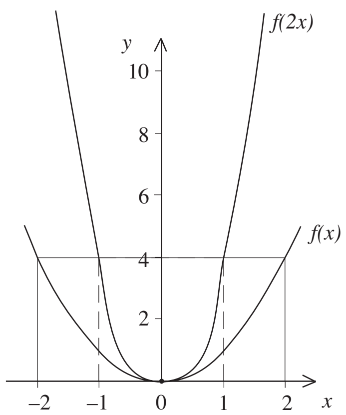


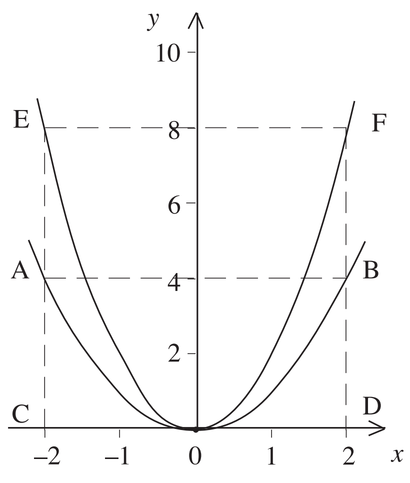
The graph of $y = f(x)$ is mapped onto $f(x+1)$ by a translation of 1 unit to the left.

> In general $f(x+a)$ translates a curve a units to the left and $f(x-a)$ translates a curve a units to the right, where a is a positive number.



The curve for $f(2x)$ is much steeper than for $f(x)$. This is because the curve has been compressed by a factor of 2 in the x-direction. Compare the rectangles ABCD and EFGH.

> In general the curve of $y = f(kx)$ will be compressed by a factor of k in the x-direction, where $k > 1$.



Here the curve $y = f(x)$ has been stretched by a factor of 2 in the vertical or y-direction to obtain the curve $y = 2f(x)$. Compare the rectangles ABCD and CDFE.

> In general the curve of $y = kf(x)$ stretches the graph of $y = f(x)$ by a factor of k in the y-direction where $k > 1$.

Note that if k is negative and $k < -1$, the curve will be stretched and reflected in the x-axis, while if $-1 < k < 1$, it is compressed.