ANGLES AND SYMMETRY

Text

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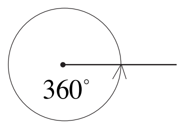
Angles and Symmetry

1 Measuring Angles

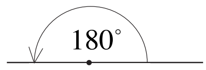
A *protractor* can be used to measure or draw angles.

Note

The angle around a complete circle is 360°.

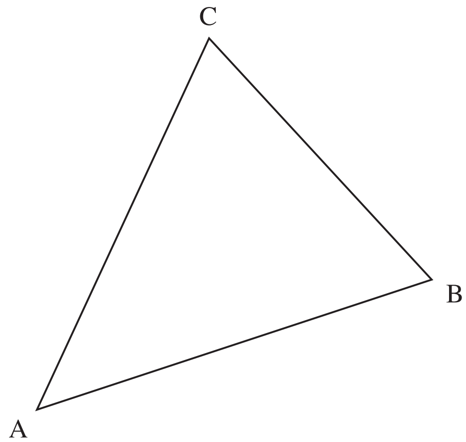


The angle around a point on a straight line is 180°.



Worked Example 1

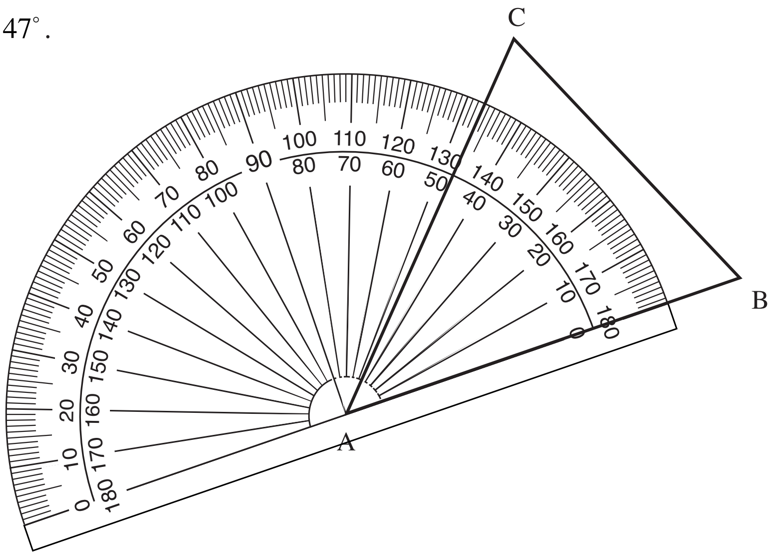
Measure the angle CAB in the triangle shown.



Solution

Place a protractor on the triangle as shown.

The angle is measured as 47°.

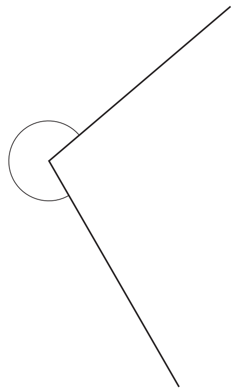


Note

When measuring an angle, start from the 0° which is in line with an arm of the angle.

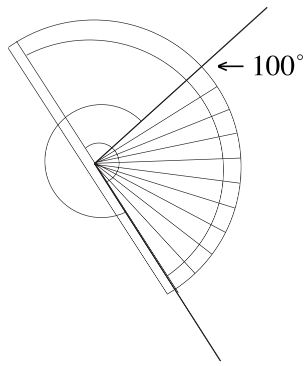
Worked Example 2

Measure the marked angle.



Solution

Using a protractor, the smaller angle is measured as 100°.



So

$$\text{required angle} = 360^\circ - 100^\circ$$
$$= 260^\circ$$

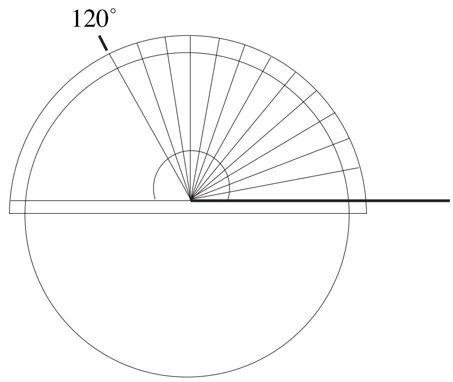
Worked Example 3

Draw angles of

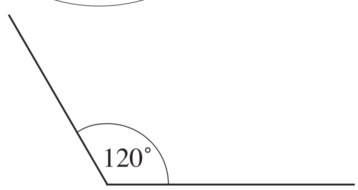
(a) 120° (b) 330°.

Solution

(a) Draw a horizontal line.
Place a protractor on top of the line and draw a mark at 120°.



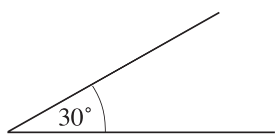
Then remove the protractor and draw the angle.



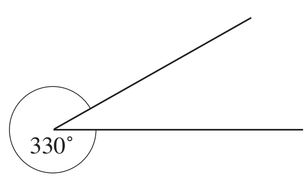
(b) To draw the angle of 330°, first subtract 330° from 360°:

$$360^\circ - 330^\circ = 30^\circ$$

Draw an angle of 30°.



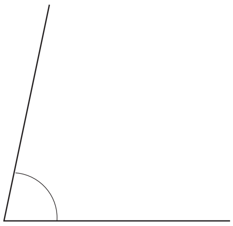
The larger angle will be 330°.



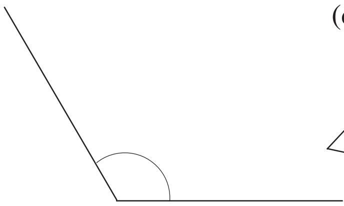
Exercises

1. Estimate the size of each angle, then measure it with a protractor.

(a)



(b)



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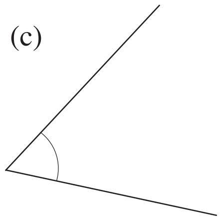

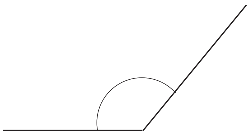

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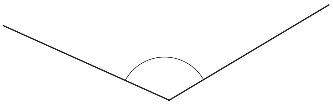

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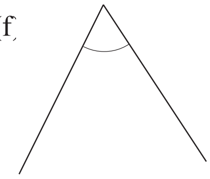
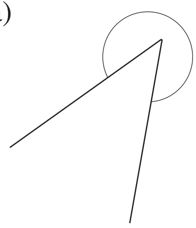


2. Draw angles with the following sizes.

(a)	50°	(b)	70°	(c)	82°
(d)	42°	(e)	80°	(f)	100°
(g)	140°	(h)	175°	(i)	160°

3. Measure these angles.

(a)



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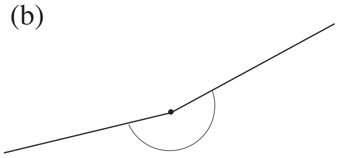

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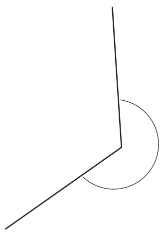

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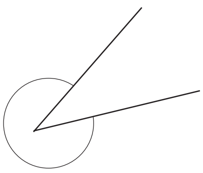

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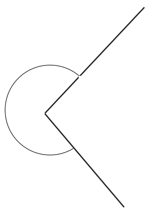

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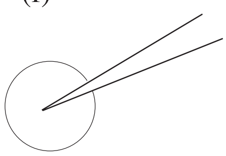
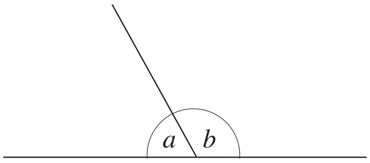


4. Draw angles with the following sizes.

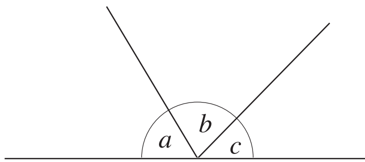
(a)	320°	(b)	190°	(c)	260°
(d)	210°	(e)	345°	(f)	318°

5. Measure each named (a, b, c) angle below and add up the angles in each diagram. What do you notice?

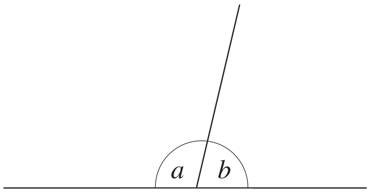
(a)



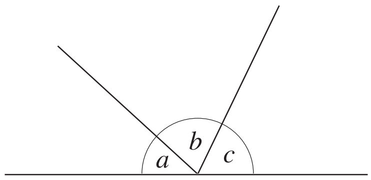
(b)



(c)

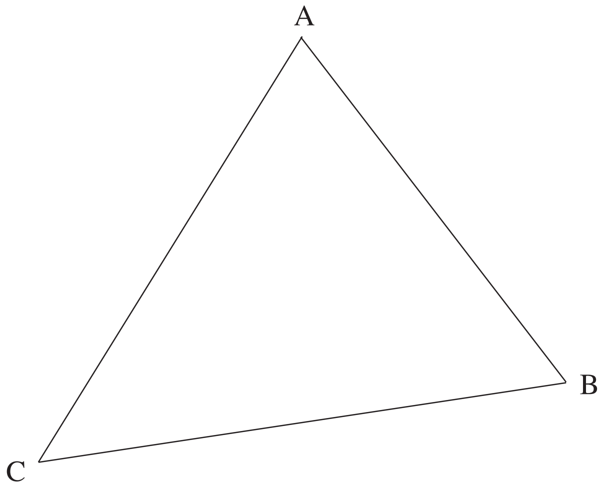


(d)

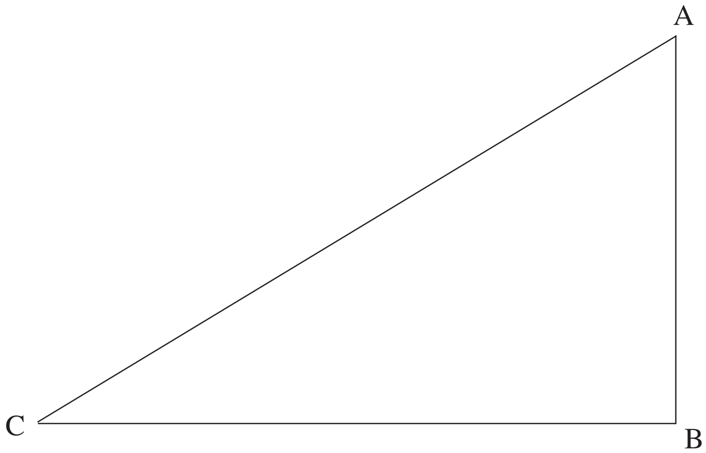


6. For each triangle below, measure each interior angle and add up the three angles you obtain.

(a)



(b)



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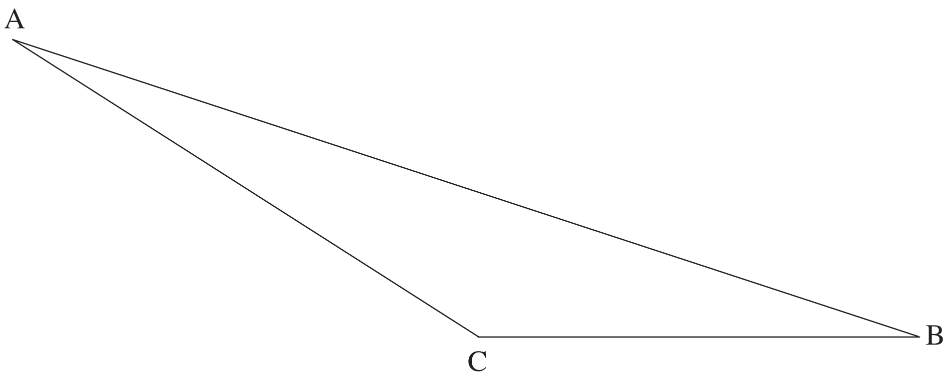

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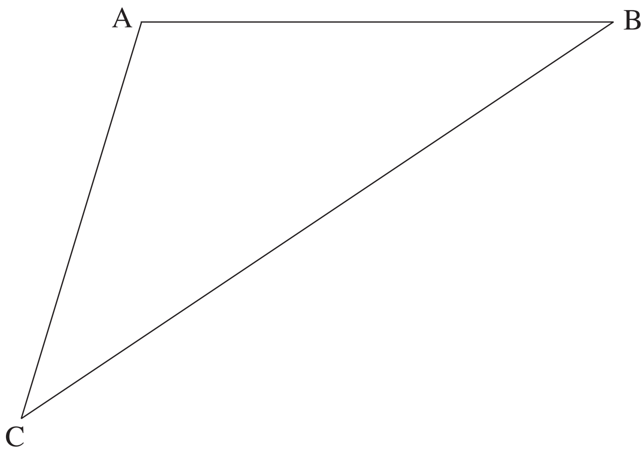
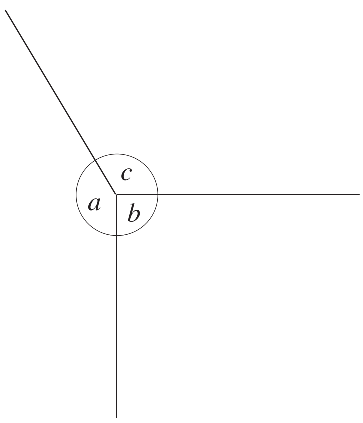


Do you obtain the same final result in each case?

7. In each diagram below, measure the angles marked with letters and find their total.

What do you notice about the totals?

(a)



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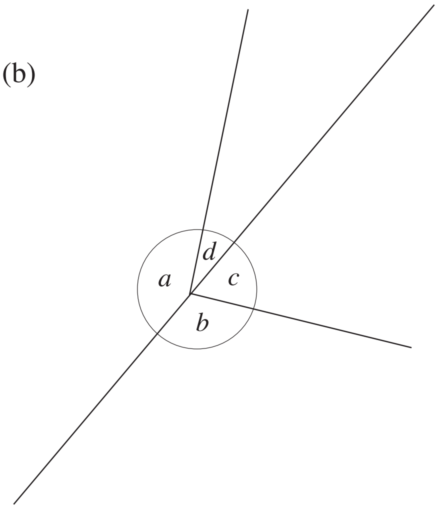

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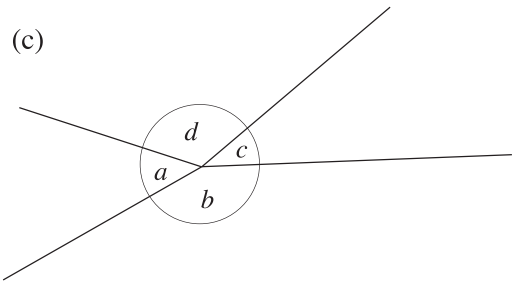
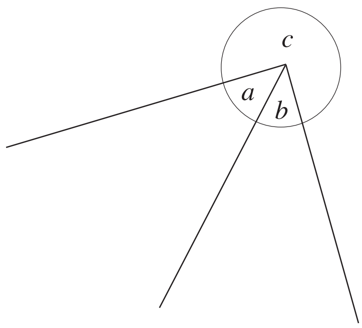
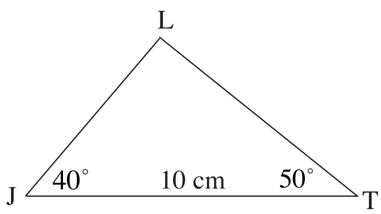


(d)



8. (a) Draw a straight line JK that is 10 cm long.

(b) Draw angles of 40° and 50° at J and K respectively, to form the triangle JKL shown in the diagram.

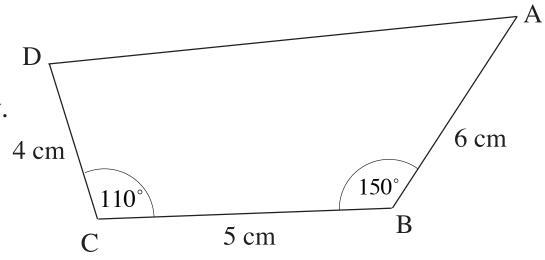


(c) Measure the lengths of JL and KL and the size of the remaining angle.

9. The diagram shows a rough sketch of a quadrilateral.

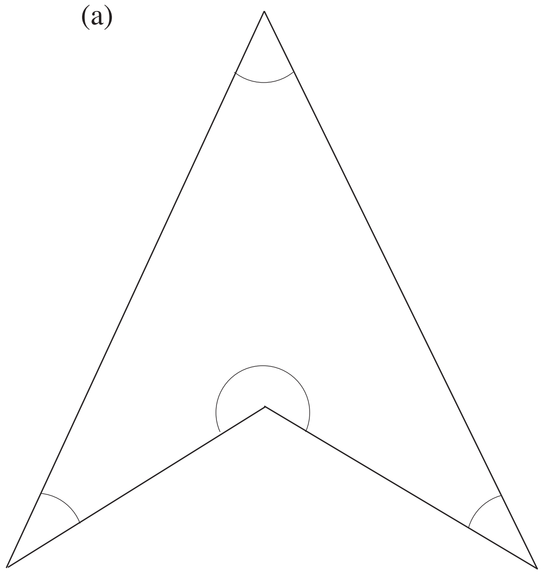
   (a) Draw the quadrilateral accurately.

   (b) Measure the length of DA and the size of the other two angles.

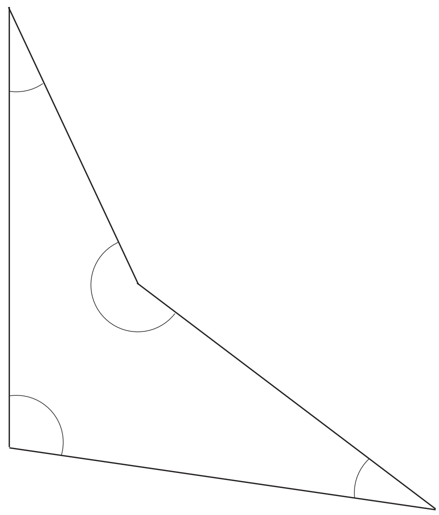


10. Measure the *interior* (inside) angles of these quadrilaterals.
    In each case find the total sum of the angles. What do you notice?

(a)



(b)



11. Draw two different pentagons.

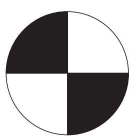
   (a) Measure each of the angles in both pentagons.

   (b) Add up your answers to find the total of the angles in each pentagon.

   (c) Do you think that the angles in a pentagon will always add up to the same number?

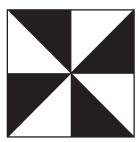
2 Line and Rotational Symmetry

An object has *rotational symmetry* if it can be rotated about a point so that it fits on top of itself without completing a full turn. The shapes below have rotational symmetry.



In a complete turn this shape fits on top of itself two times.

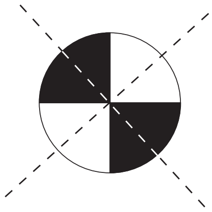
It has rotational symmetry of order 2.



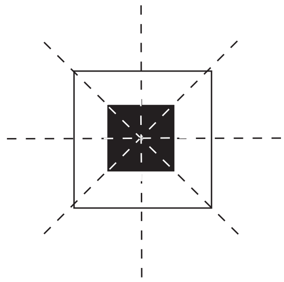
In a complete turn this shape fits on top of itself four times.

It has rotational symmetry of order 4.

Shapes have *line symmetry* if a mirror could be placed so that one side is an exact reflection of the other. These imaginary 'mirror lines' are shown by dotted lines in the diagrams below.



This shape has
2 lines of symmetry.



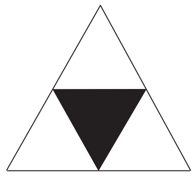
This shape has
4 lines of symmetry.

Worked Example 1

For the given shape, state:

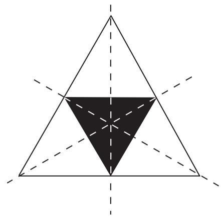
(a) the number of lines of symmetry,

(b) the order of rotational symmetry.

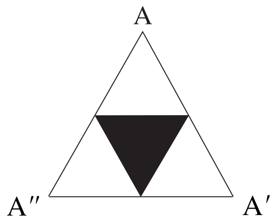


Solution

(a) There are 3 lines of symmetry as shown.



(b) There is rotational symmetry with order 3, because the point marked A could be rotated to A' then to A'' and fit exactly over its original shape at each of these points.



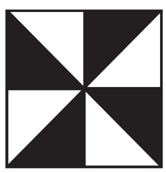
Exercises

1. Which of the shapes below have

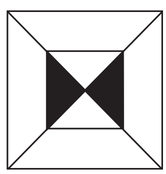
 (a) line symmetry (b) rotational symmetry?

 For line symmetry, copy the shape and draw in the mirror lines.
 For rotational symmetry state the order.

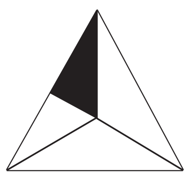
A



B



C



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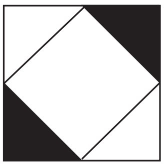
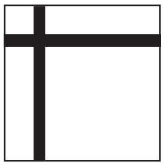


E


F


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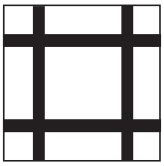

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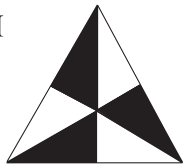

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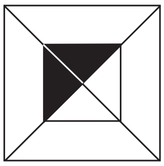
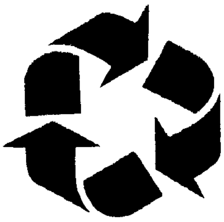


2. For each shape below state:

   (a) whether the shape has any symmetry;

   (b) how many lines of symmetry it has;

   (c) the order of symmetry if it has rotational symmetry.

A


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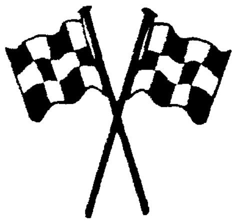
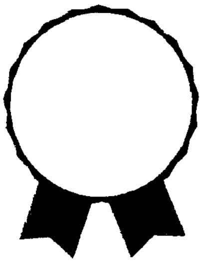


C


D


E


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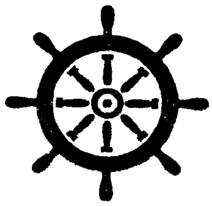

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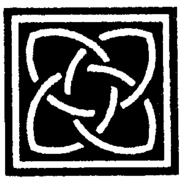
H


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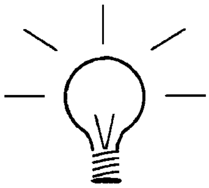
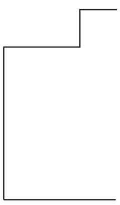


3. Copy and complete each shape below so that it has line symmetry but not rotational symmetry. Mark clearly the lines of symmetry.

(a)


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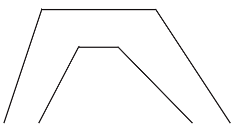

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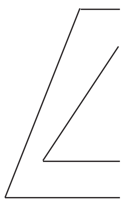
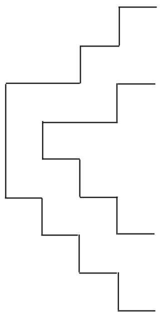
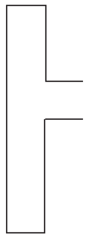
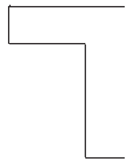
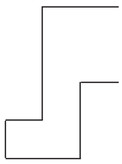
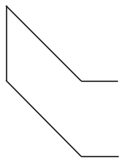
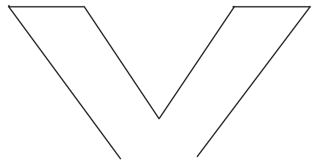
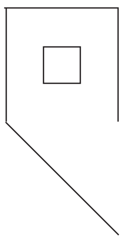
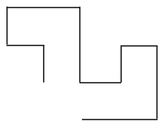
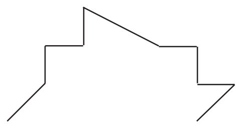


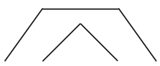
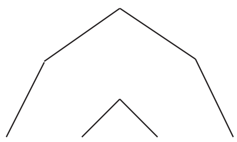
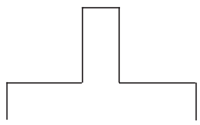
(d)  (e)  (f) 

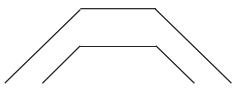
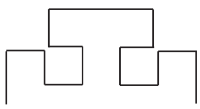
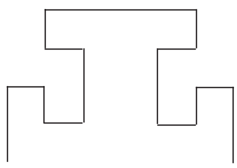
4. Copy and, if possible, complete each shape below, so that they have rotational symmetry, but not line symmetry. In each case state the order of the rotational symmetry.

(a)  (b)  (c) 

(d)  (e)  (f) 

5. Copy and complete each of the following shapes, so that they have both rotational and line symmetry. In each case draw the lines of symmetry and state the order of the rotational symmetry.

(a)  (b)  (c) 

(d)  (e)  (f) 

6. Draw a square and show all its lines of symmetry.

7. (a) Draw a triangle with:

   (i) 1 line of symmetry (ii) 3 lines of symmetry.

   (b) Is it possible to draw a triangle with 2 lines of symmetry?

8. Draw a shape which has 4 lines of symmetry.

9. Draw a shape with rotational symmetry of order:

(a) 2 (b) 3 (c) 4 (d) 5

10. Can you draw:

(a) a pentagon with exactly 2 lines of symmetry,

(b) a hexagon with exactly 2 lines of symmetry,

(c) an octagon with exactly 3 lines of symmetry?

11. These are the initials of the International Association of Whistlers.

I A W

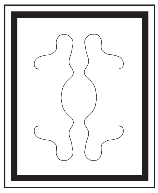
Which of these letters has rotational symmetry?

12. Which of the designs below have line symmetry?

(a)

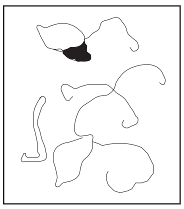
Taj Mahal floor tile

(b)



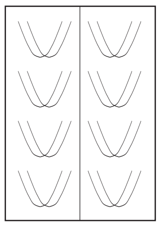
Asian carpet design

(c)



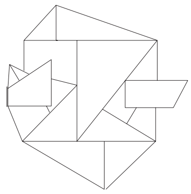
Contemporary art

(d)



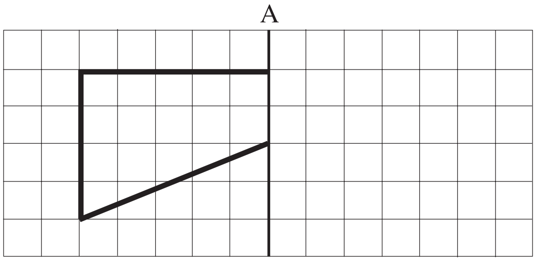
Wallpaper pattern

(e)

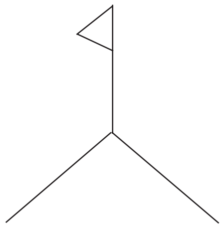


Tile design

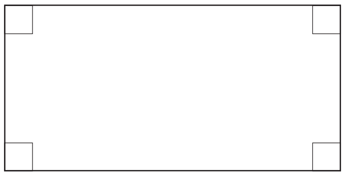
13. (a) Copy and draw the reflection of this shape in the mirror line AB.



(b) Copy and complete the diagram opposite so that it has rotational symmetry.



(c) What is the order of rotational symmetry of this shape?



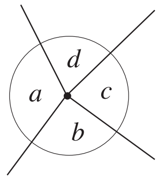
3 Angle Geometry

There are a number of important results concerning angles in different shapes, at a point and on a line. In this section the following results will be used.

1. *Angles at a Point*

   The angles at a point will always add up to 360°.

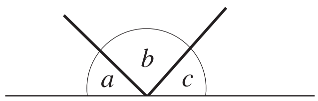
   It does not matter how many angles are formed at the point – their total will always be 360°.



$a + b + c + d = 360^\circ$

2. *Angles on a Line*

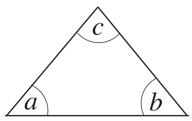
   Any angles that form a straight line add up to 180°.



$a + b + c = 180^\circ$

3. *Angles in a Triangle*

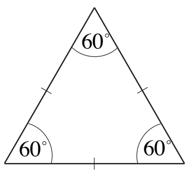
   The angles in any triangle add up to 180°.



$a + b + c = 180^\circ$

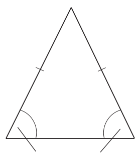
4. *Angles in an Equilateral Triangle*

   In an equilateral triangle all the angles are 60° and all the sides are the same length.



5. *Angles in an Isosceles Triangle*

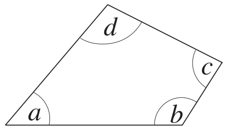
   In an isosceles triangle two sides are the same length and two angles are the same size.



equal angles

6. *Angles in a Quadrilateral*

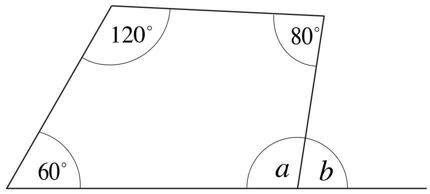
   The angles in any quadrilateral add up to 360°.



$a + b + c + d = 360^\circ$

Worked Example 1

Find the sizes of angles a and b in the diagram below.



Solution

First consider the quadrilateral. All the angles of this shape must add up to $360°$, so

$$\begin{aligned} 60° + 120° + 80° + a &= 360° \\ 260° + a &= 360° \\ a &= 360° - 260° \\ &= 100° \end{aligned}$$

Then consider the straight line formed by the angles a and b. These two angles must add up to $180°$ so,

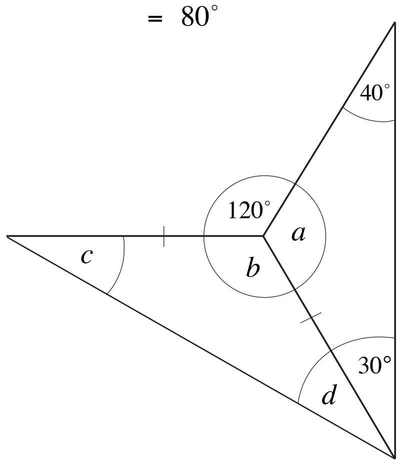
$$a + b = 180°$$

but $a = 100°$, so

$$\begin{aligned} 100° + b &= 180° \\ b &= 180° - 100° \\ &= 80° \end{aligned}$$

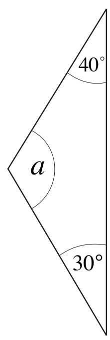
Worked Example 2

Find the angles a, b, c and d in the diagram.



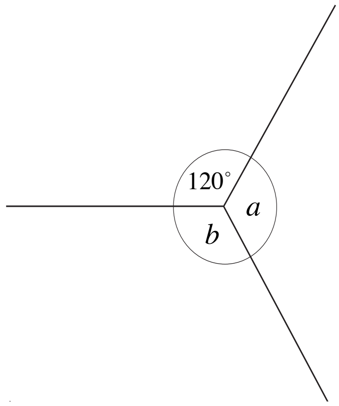
Solution

First consider the triangle shown.
The angles of this triangle must add up to $180°$,



So, $$40° + 30° + a = 180°$$

Next consider the angles round the point shown.



The three angles must add up to 360°, so

$$120^\circ + b + a = 360^\circ$$

but $a = 110^\circ$, so

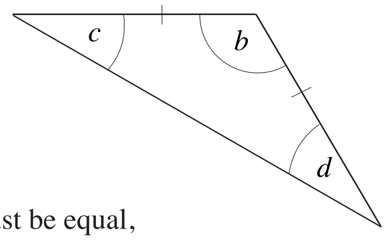
$$120^\circ + 110^\circ + b = 360^\circ$$

$$230^\circ + b = 360^\circ$$

$$b = 360^\circ - 230^\circ$$

$$= 130^\circ$$

Finally, consider the second triangle.



The angles must add up to 180°, so

$$c + b + d = 180^\circ$$

As this is an isosceles triangle the two angles, c and d, must be equal, so using $c = d$ and the fact that $b = 130^\circ$, gives

$$c + 130^\circ + c = 180^\circ$$

$$2c = 180^\circ - 130^\circ$$

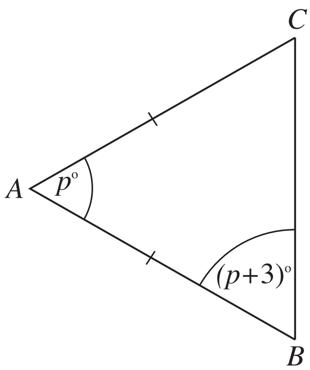
$$= 50^\circ$$

$$c = 25^\circ$$

As $c = 25^\circ$, $d = 25^\circ$.

Worked Example 3

In the figure below, **not drawn to scale**, ABC is an isosceles triangle with $\angle CAB = p^\circ$ and $\angle ABC = (p + 3)^\circ$.



(a) Write an expression in terms of p for the value of the angle at C.

(b) Determine the size of EACH angle in the triangle.

Solution

(a) As *ABC* is an isosceles triangle,

$$\angle ACB = p + 3^\circ$$

(b) For triangle *ABC*,

$$p + (p + 3) + (p + 3) = 180^\circ$$

$$3p + 6^\circ = 180^\circ \qquad \text{(take 6 from each side)}$$

$$3p = 180^\circ - 6^\circ$$

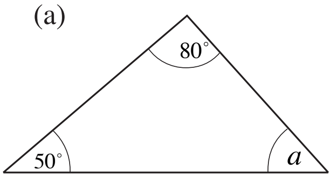
$$3p = 174^\circ \qquad \text{(divide both sides by 3)}$$

$$p = 58^\circ$$

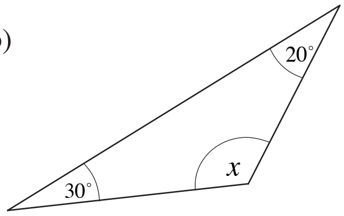
Exercises

1. Find the size of the angles marked with a letter in each diagram.

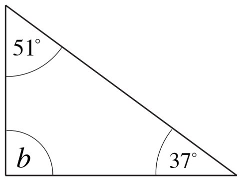
(a)



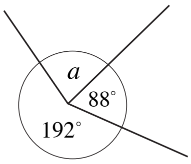
(b)



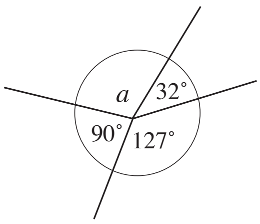
(c)



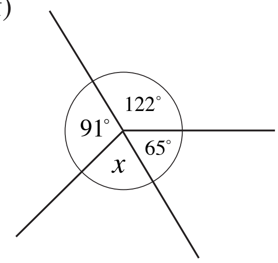
(d)



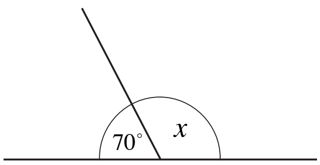
(e)



(f)



(g)



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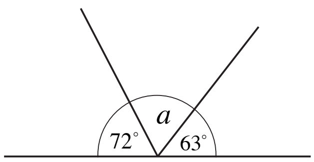
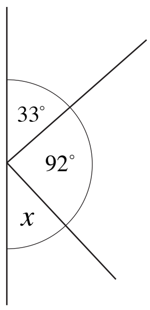
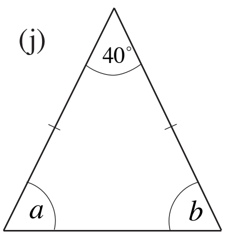


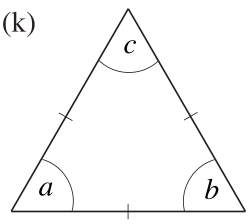
(i)



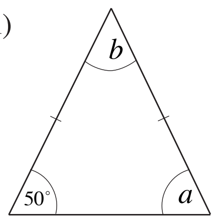
(j)



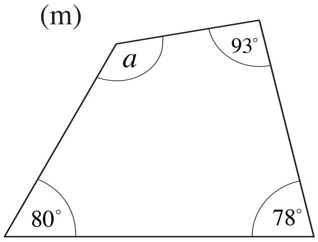
(k)



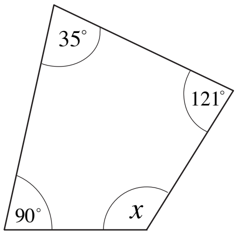
(l)



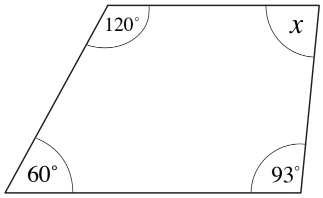
(m)



(n)

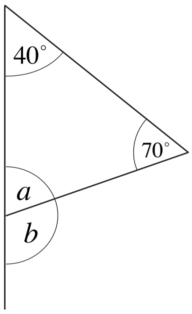


(o)

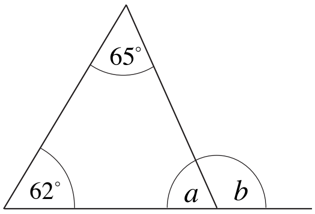


2. (a) For each triangle, find the angles marked a and b .

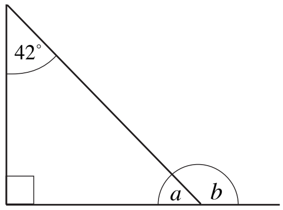
(i)



(ii)



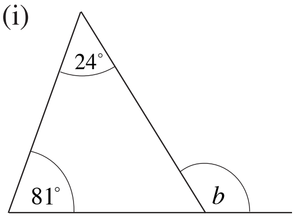
(iii)



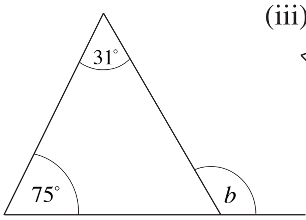
(b) What do you notice about the angle marked b and the other two angles given in each problem?

(c) Find the size of the angle b in each problem below without working out the size of any other angles.

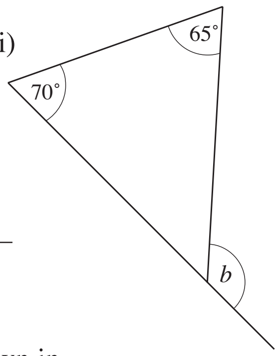
(i)



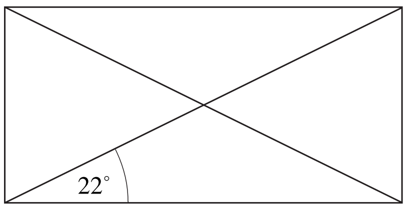
(ii)



(iii)



3. The diagram below shows a rectangle with its diagonals drawn in.



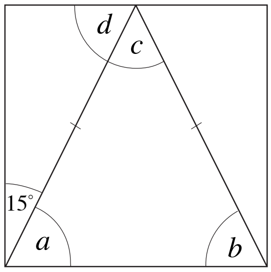
(a) Copy the diagram and mark in all the other angles that are 22°.

(b) Find the sizes of all the other angles.

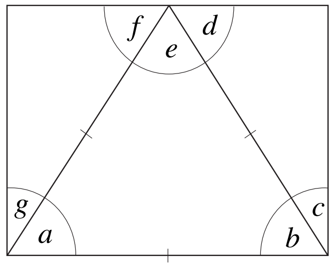
4. Find the angles marked with letters in each of the following diagrams.

In each diagram the lines all lie inside a rectangle.

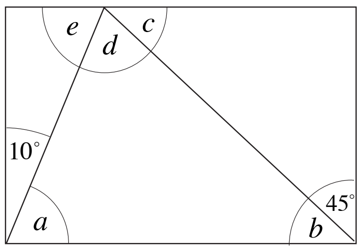
(a)



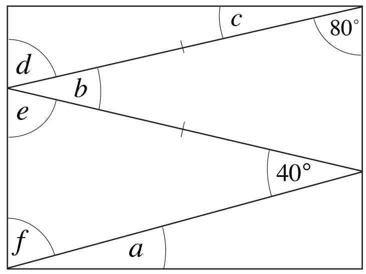
(b)



(c)

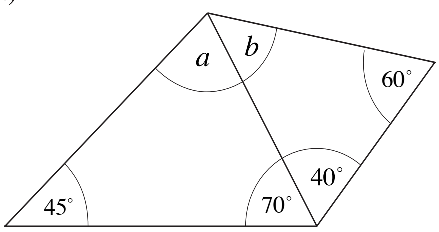


(d)

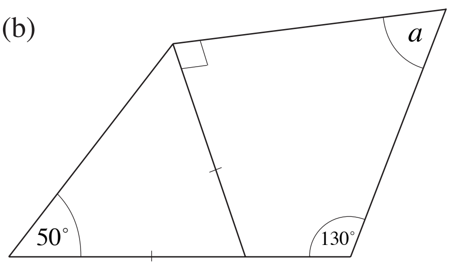


5. Find the angles marked with letters in each quadrilateral below.

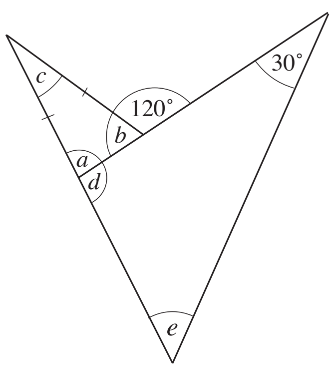
(a)



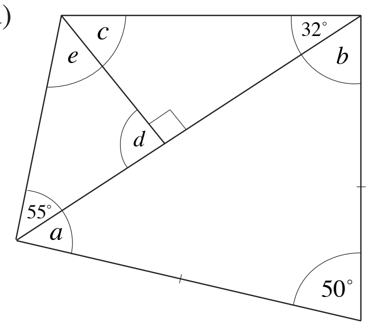
(b)



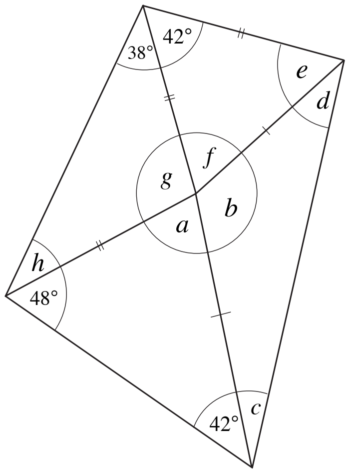
(c)



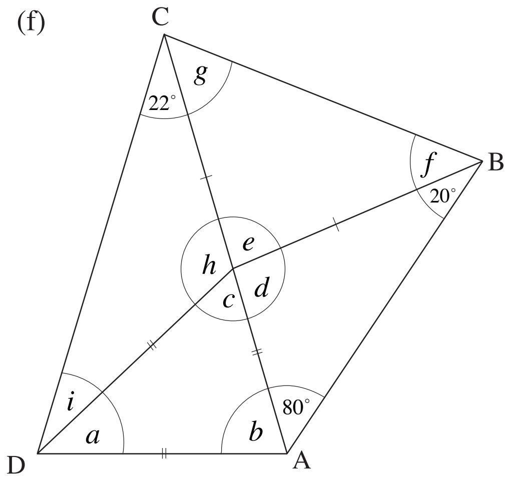
(d)



(e)

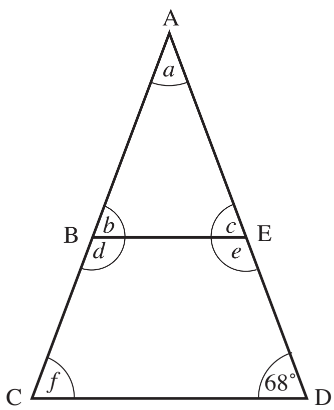


(f)



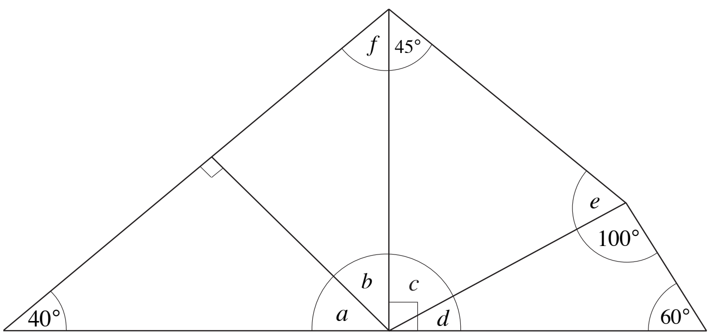
AC is a straight line.

6. A swing is built from two metal frames. A side view of the swing is shown below.



The lengths of AB and AE of the swing are the same and the lengths of AC and AD of the swing are the same. Find the sizes of the angles *a*, *b*, *c*, *d*, *e* and *f*.

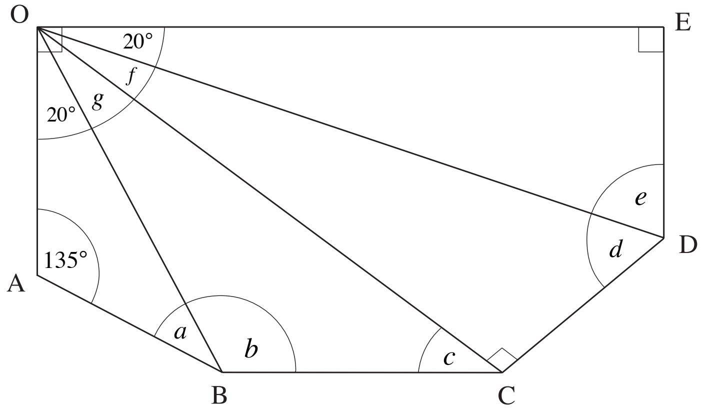
7. The diagram shows a wooden frame that forms part of the roof of a house.



Find the sizes of the angles *a*, *b*, *c*, *d*, *e* and *f*.

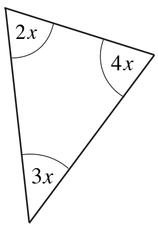
8. The diagram shows the plan for a conservatory. Lines are drawn from the point O to each of the other corners. Find all the angles marked with letters, if

$$A\hat{B}C = B\hat{C}D = C\hat{D}E = 135°$$

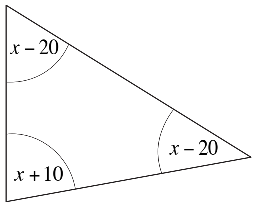


9. Write down an equation and use it to find the value of x in each diagram.

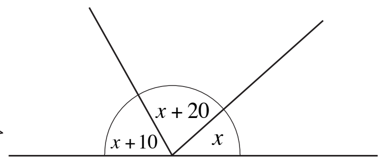
(a)



(b)



(c)



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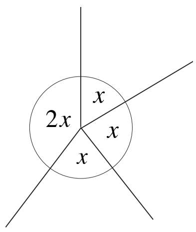
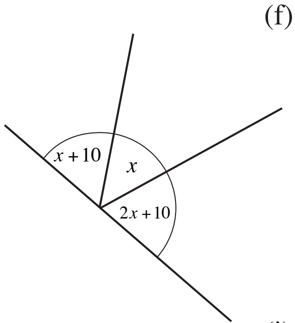
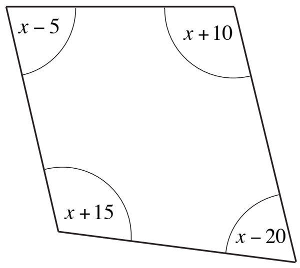


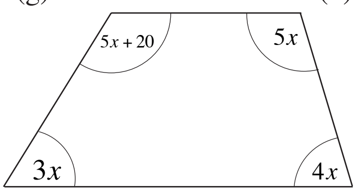
(e)



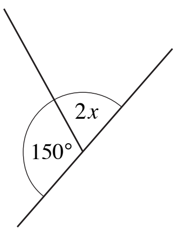
(f)



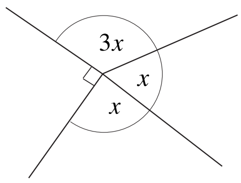
(g)



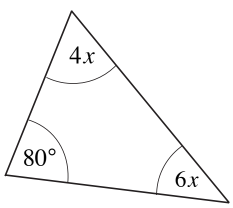
(h)



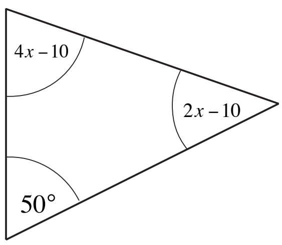
(i)



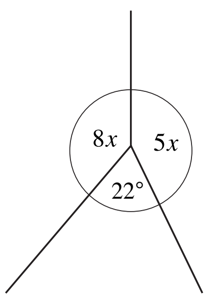
(j)



(k)



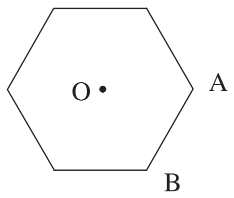
(l)



10. The diagram shows a regular hexagon.

O is the point at the centre of the hexagon.

A and B are two vertices.



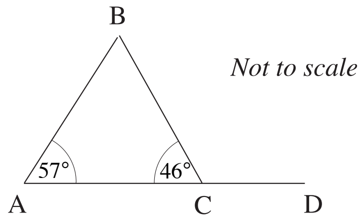
(a) Write down the order of rotational symmetry of the regular hexagon.

(b) Draw the lines from O to A and from O to B.

(i) Write down the size of angle AOB.

(ii) Write down the mathematical name for triangle AOB.

11. Calculate angles BCD and ABC, giving reasons for your answers.



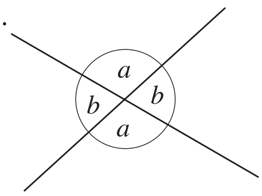
4 Angles with Parallel and Intersecting Lines

Opposite Angles

When any two lines intersect, two pairs of equal angles are formed.

The two angles marked a are a pair of *opposite* equal angles.

The angles marked b are also a pair of opposite equal angles.



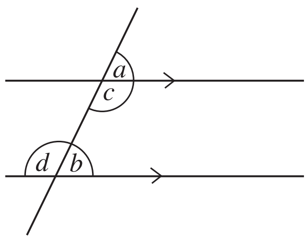
Corresponding Angles

When a line intersects a pair of parallel lines, $a = b$.

The angles a and b are called *corresponding* angles.

Alternate Angles

The angles c and d are equal.

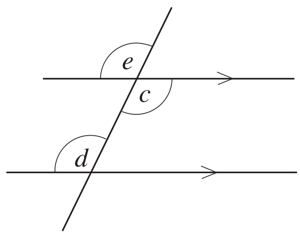


Proof

This result follows since c and e are opposite angles, so $c = e$, and e and d are corresponding angles, so $c = d$.

Hence $c = e = d$

The angles c and d are called *alternate* angles.



Supplementary Angles

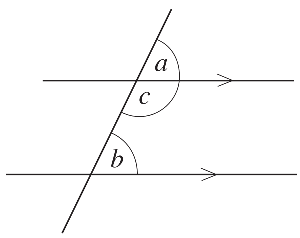
The angles b and c add up to $180°$.

Proof

This result follows since $a + c = 180°$ (straight line), and $a = b$ since they are corresponding angles.

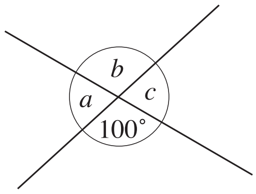
Hence $b + c = 180°$.

These angles are called *supplementary* angles.



Worked Example 1

Find the angles marked a, b and c.



Solution

There are two pairs of opposite angles here so:

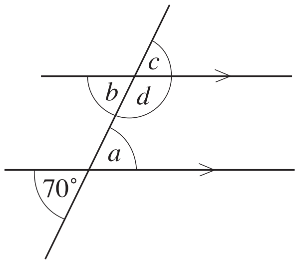
$$b = 100 \text{ and } a = c$$

Also a and b form a straight line so

$$\begin{aligned} a + b &= 180° \\ a + 100° &= 180° \\ a &= 80°, \text{ so } c = 80° \end{aligned}$$

Worked Example 2

Find the sizes of the angles marked a, b, c and d in the diagram.



Solution

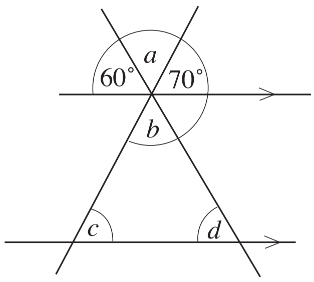
First note the two parallel lines marked with arrow heads.

Then find a. The angle a and the angle marked $70°$ are *opposite* angles, so $a = 70°$.
The angles a and b are *alternate* angles so $a = b = 70°$.
The angles b and c are *opposite* angles so $b = c = 70°$.
The angles a and d are a pair of *interior* angles, so $a + d = 180°$, but $a = 70°$, so

$$\begin{aligned} 70° + d &= 180° \\ d &= 180° - 70° \\ &= 110° \end{aligned}$$

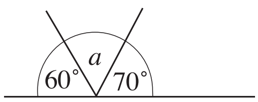
Worked Example 3

Find the angles marked a, b, c and d in the diagram.



Solution

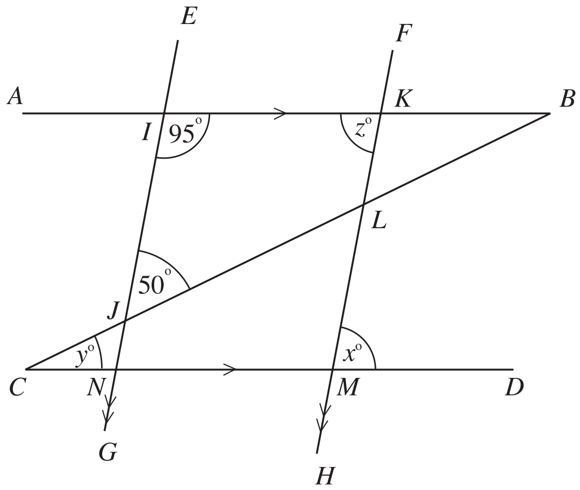
To find the angle a, consider the three angles that form a straight line. So



$$\begin{aligned} 60° + a + 70° &= 180° \\ a &= 180° - 130° \\ &= 50° \end{aligned}$$

The angle marked b is opposite the angle a, so $b = a = 50°$.
Now c and d can be found using *corresponding* angles.
The angle c and the $70°$ angle are corresponding angles, so $c = 70°$.
The angle d and the $60°$ angle are corresponding angles, so $d = 60°$.

Worked Example 4



In the diagram above, **not drawn to scale**, *AB* is parallel to *CD* and *EG* is parallel to *FH*, angle *IJL* = 50° and angle *KIJ* = 95°.

Calculate the values of *x, y* and *z,* showing clearly the steps in your calculations.

Solution

<u>Value of *x*</u>

Angles *BIG* and *END* are supplementary angles, so

$$95° + E\hat{N}D = 180°$$

$$E\hat{N}D = 180° - 95°$$

$$\text{i.e.} \quad E\hat{N}D = 85°$$

But angles *END* and *FMD* are corresponding angles, so

$$85° = x$$

<u>Value of *y*</u>

Angles *BCD* (*y*) and *ABC* are alternate angles, so

$$y = A\hat{B}C$$

In triangle *BIJ*,

$$y + 95° + 50° = 180°$$

$$y = 180° - (95 + 50)°$$

$$= 180° - 145°$$

$$\text{i.e.} \quad y = 35°$$

<u>Value of *z*</u>

Angles $A\hat{K}H$ (*z*) and *FMD* (*x*) are alternate angles, so

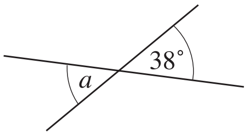
$$z = x°$$

$$\text{i.e.} \quad z = 85°$$

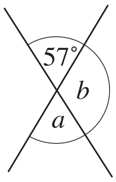
Exercises

1. Find the angles marked in each diagram, *giving reasons for your answers*.

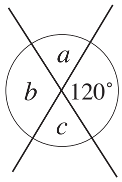
(a)



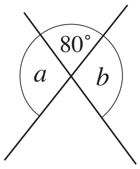
(b)



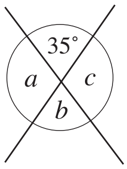
(c)



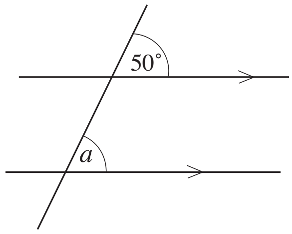
(d)



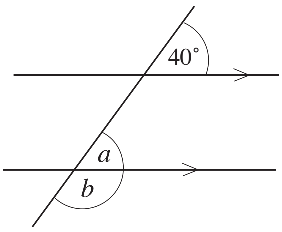
(e)



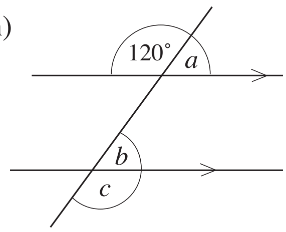
(f)



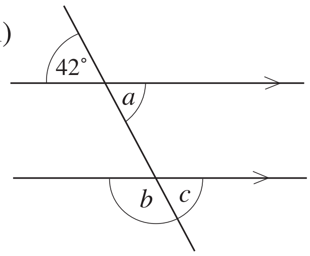
(g)



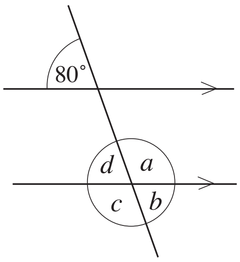
(h)



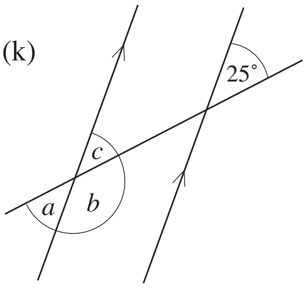
(i)



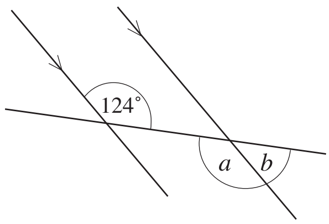
(j)



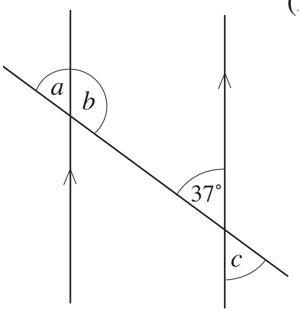
(k)



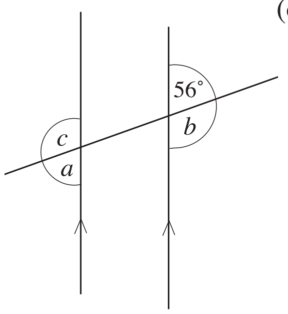
(l)



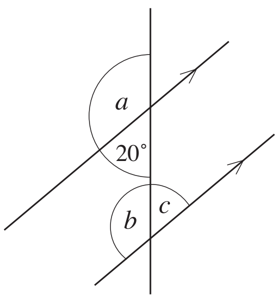
(m)



(n)

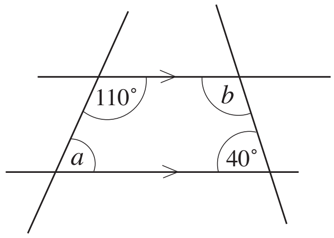


(o)

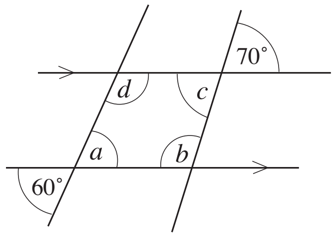


2. Find the size of the angles marked *a*, *b*, *c*, etc. in each of the diagrams below.

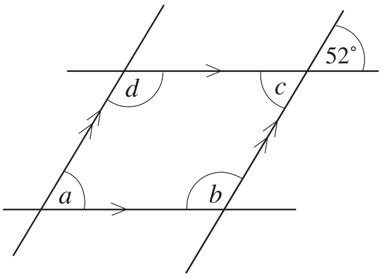
(a)



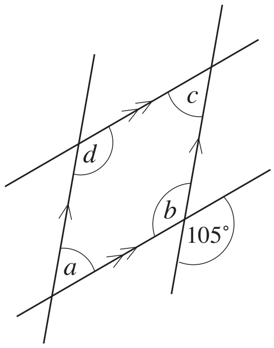
(b)



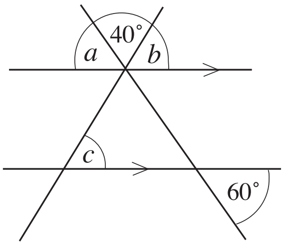
(c)



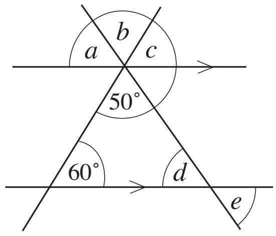
(d)



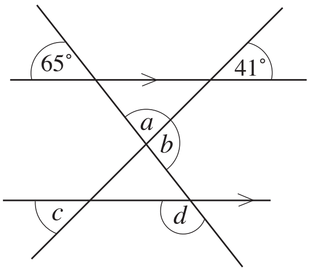
(e)



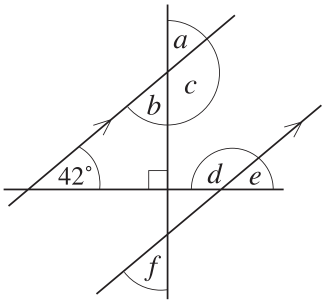
(f)



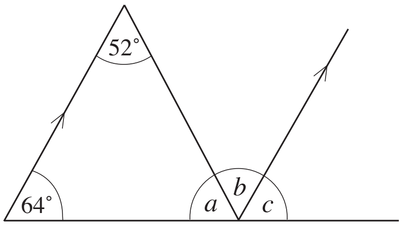
(g)



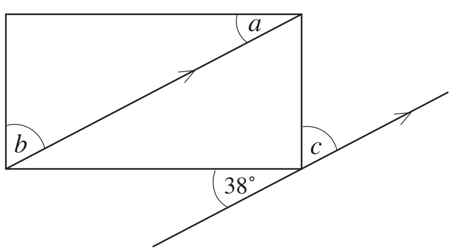
(h)



(i)

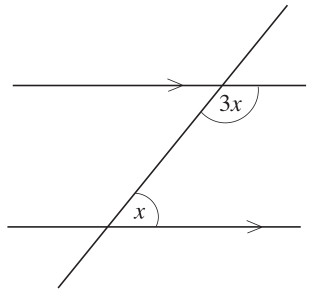


(j)

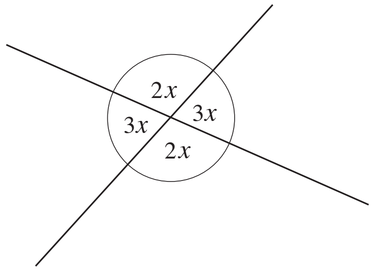


3. By considering each diagram, write down an equation and find the value of x.

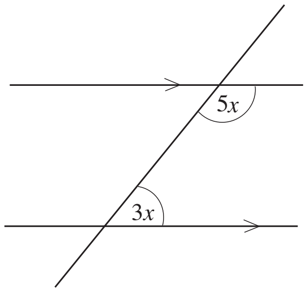
(a)



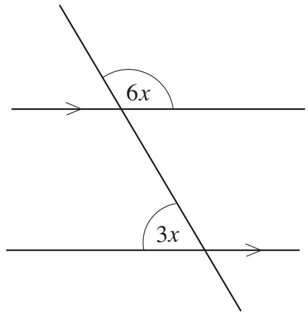
(b)



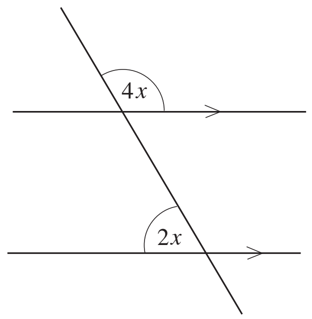
(c)



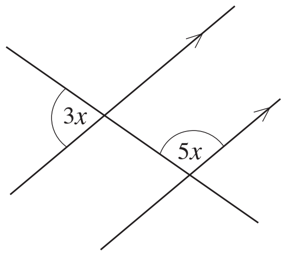
(d)



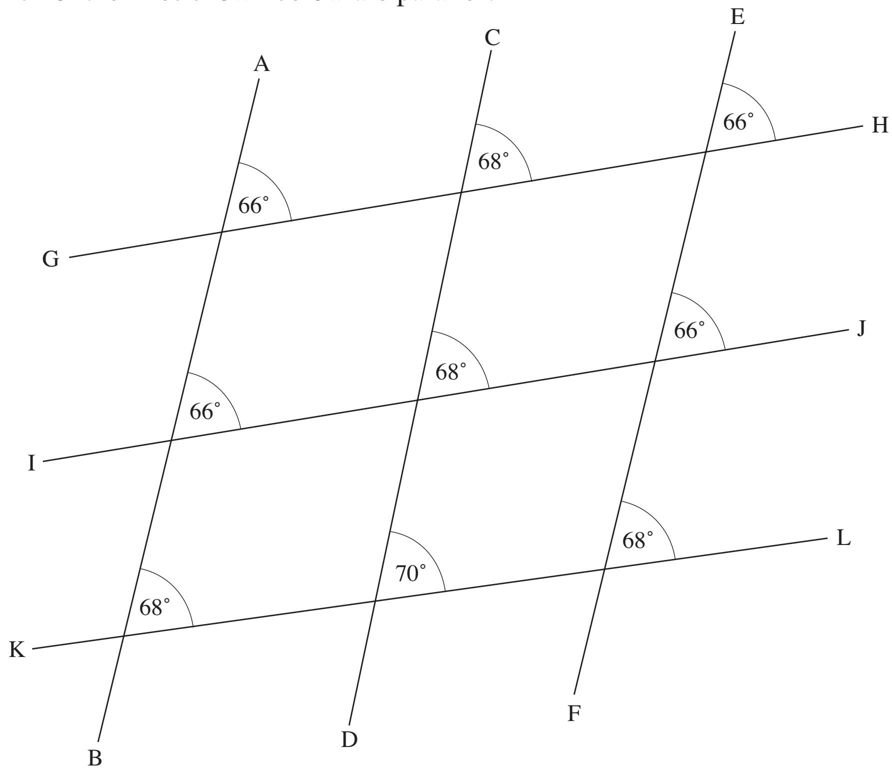
(e)



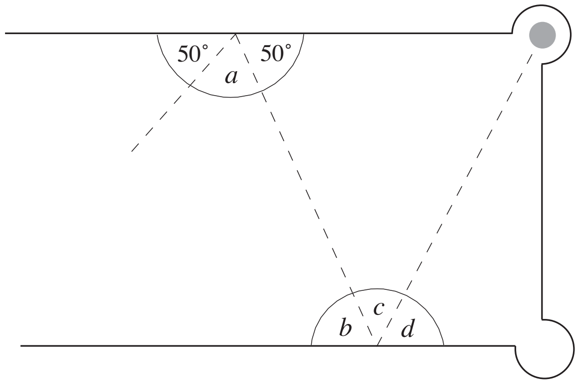
(f)



4. Which of the lines shown below are parallel?



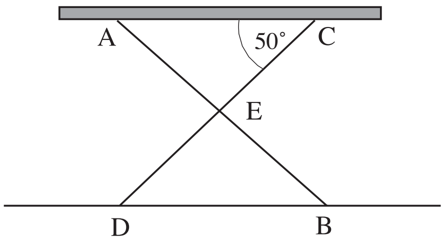
5. The diagram shows the path of a pool ball as it bounces off cushions on opposite sides of a pool table.



(a) Find the angles *a* and *b*.

(b) If, after the second bounce, the path is parallel to the path before the first bounce, find *c* and *d*.

6. A workbench is standing on a horizontal floor. The side of the workbench is shown.



The legs AB and CD are equal in length and joined at E. AE = EC

(a) Which two lines are parallel?

Angle ACD is $50°$.

(b) Work out the size of angle BAC giving a reason for your answer.

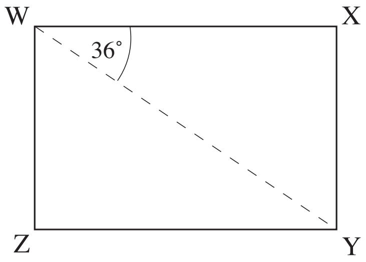
7. Here are the names of some quadrilaterals.

Square
Rectangle
Rhombus
Parallelogram
Trapezium
Kite

(a) Write down the names of the quadrilaterals which have two pairs of parallel sides.

(b) Write down the names of the quadrilaterals which must have two pairs of equal opposite sides.

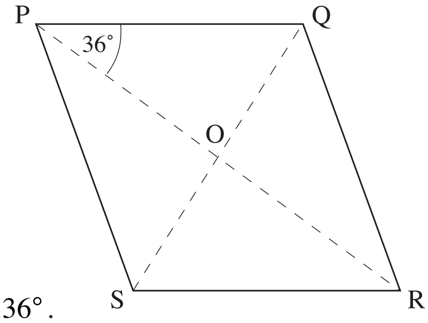
8. WXYZ is a rectangle.



Not to scale

(a) Angle XWY = 36°.
Work out the size of angle WYZ, giving a reason for your answer.

PQRS is a rhombus.



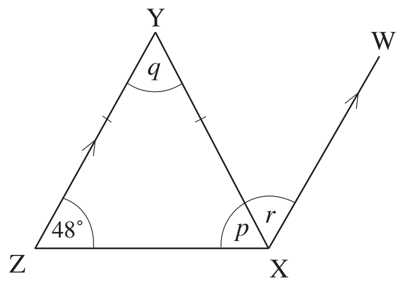
Not to scale

(b) Angle QPR = 36°.

The diagonals PR and QS intersect at O.

Work out the size of angle PQS, giving a reason for your answer.

9. In the diagram, XY = ZY and ZY is parallel to XW.



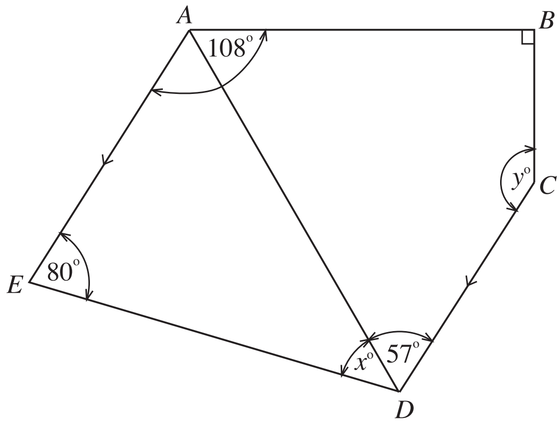
Not to scale

(a) Write down the size of angle p.

(b) Calculate the size of angle q. Give a reason for your answer.

(c) Give a reason why angle q = angle r.

10. In the diagram shown below, $ABCDE$ is a pentagon. $\angle BAE = 108°$, $\angle ABC = 90°$, $\angle AED = 80°$, $\angle ADC = 57°$ and AE is parallel to CD.



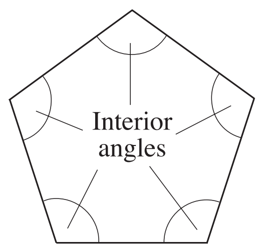
Calculate the size of the angle marked

(a) $x°$

(b) $y°$.

5 Angle Symmetry in Regular Polygons

Regular polygons will have both line and rotational symmetry. This symmetry can be used to find the *interior* angles of a regular polygon.

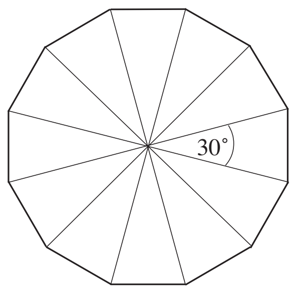


Worked Example 1

Find the interior angle of a regular dodecagon.

Solution

The diagram shows how a regular dodecagon can be split into 12 isosceles triangles.



As there are $360°$ around the centre of the dodecagon, the centre angle in each triangle is

$$\frac{360°}{12} = 30°$$

So the other angles of each triangle will together be

$$180° - 30° = 150°$$

Therefore each of the other angles will be

$$\frac{150°}{2} = 75°$$

As two adjacent angles are required to form each interior angle of the dodecagon, each interior angle will be

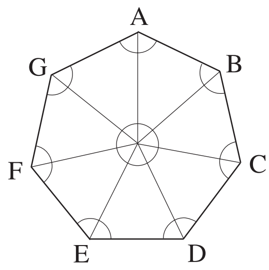
$$75° \times 2 = 150°$$

As there are 12 interior angles, the sum of these angles will be $12 \times 150° = 1800°$.

Worked Example 2

Find the sum of the interior angles of a regular heptagon.

Solution



Split the heptagon into 7 isosceles triangles.

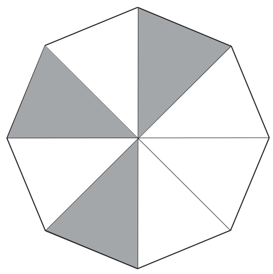
Each triangle contains three angles which add up to 180°, so the total of all the marked angles will be

$$7 \times 180° = 1260°.$$

However the angles at the point where all the triangles meet should not be included, so the sum of the interior angles is given by

$$1260° - 360° = 900°$$

Worked Example 3

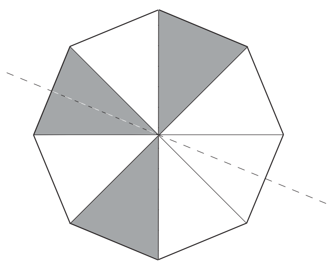


(a) Copy the octagon shown in the diagram and draw in any lines of symmetry.

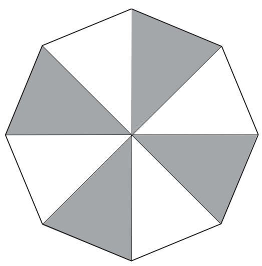
(b) Copy the octagon and shade in extra triangles so that it now has rotational symmetry.

Solution

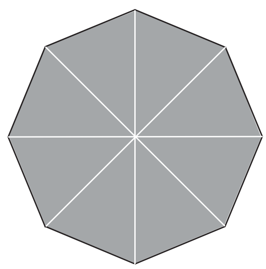
(a) There is only one line of symmetry as shown in the diagram.



(b) The original octagon has no rotational symmetry.



By shading the extra triangle shown, it has rotational symmetry of order 4.



By shading all the triangles, it has rotational symmetry of order 8.

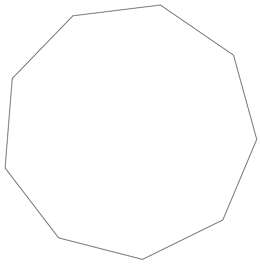
Exercises

1. Find the interior angle for a regular:

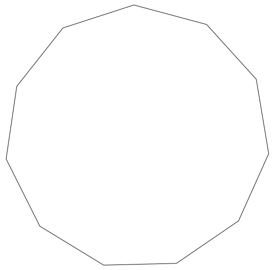
   (a) pentagon (b) hexagon
   (c) octagon (d) decagon (10 sides).

2. Find the sum of the interior angles in each polygon shown below.

   (a)



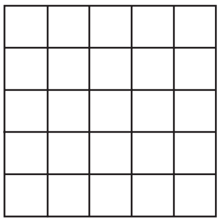
   (b)

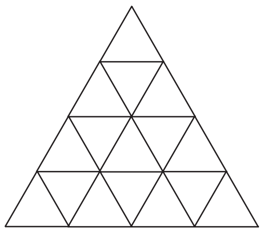


3. Which regular polygons have interior angles of:

   (a) 90° (b) 120° (c) 108°
   (d) 140° (e) 60° (f) 144° ?

4. Make 3 copies of each shape below.





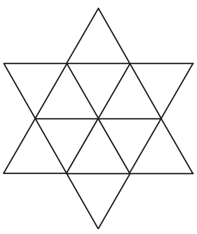
   and shade parts of them, so that:

   (a) they have line symmetry, but no rotational symmetry;
   (b) they have line symmetry and rotational symmetry;
   (c) they have rotational symmetry, but no line symmetry.

   In each case draw in the lines of symmetry and state the order of rotational symmetry.

5. (a) Draw a shape that has rotational symmetry of order 3 but no line symmetry.
   (b) Draw a shape that has rotational symmetry of order 5 but no line symmetry.

6. (a)



   For this shape, is it possible to shade smaller triangle so that is has rotational symmetry of

   (i) 2 (ii) 3 (iii) 4

   with no lines of symmetry?

(b) Is it possible to shade smaller triangles so that the shape has

(i) 1 (ii) 2 (iii) 3

lines of symmetry and no rotational symmetry?

7. (a) A polygon has 9 sides. What is the sum of the interior angles?

(b) Copy and complete the table below.

Shape	**Sum of interior angles**
Triangle	180°
Square	
Pentagon	
Hexagon	720°
Heptagon	
Octagon	

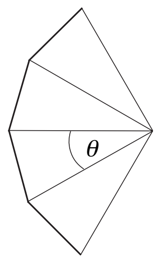
(c) Describe a rule that could be used to calculate the sum of the interior angles for a polygon with *n* sides.

(d) Find the sum of the interior angles for a 14-sided polygon.

(e) The sum of the interior angles of a polygon is $1260°$. How many sides does the polygon have?

8. (a) A regular polygon with *n* sides is split into isosceles triangles as shown in the diagram.

Find a formula for the size of the angle marked θ.



(b) Use your answer to part (a) to find a formula for the interior angle of a regular polygon with *n* sides.

(c) Use your formula to find the interior angle of a polygon with 20 sides.

9.

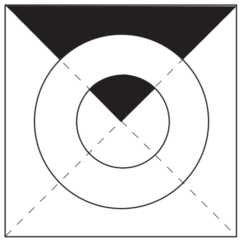
(a) Write down the order of rotational symmetry of this rectangle.

(b) Draw a shape which has rotational symmetry of order 3.

(c) (i) How many lines of symmetry has a regular pentagon?

(ii) What is the size of one exterior angle of a regular pentagon?

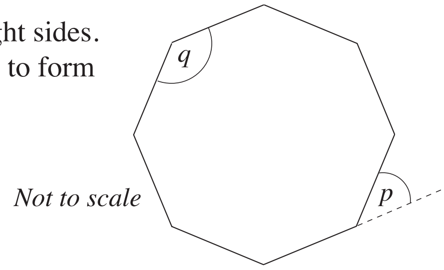
10.



The picture shows a large tile with only part of its pattern filled in.

Complete the picture so that the tile has 2 lines of symmetry and rotational symmetry of order 2.

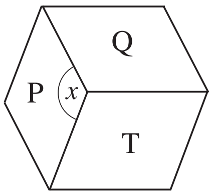
11. A regular octagon, drawn opposite, has eight sides. One side of the octagon has been extended to form angle p.



(a) Work out the size of angle p.

(b) Work out the size of angle q.

12.



The diagram shows three identical rhombuses, P, Q and T.

(a) Explain why angle x is $120°$.

(b) Rhombus Q can be rotated onto rhombus T.
   (i) Mark a centre of rotation.
   (ii) State the angle of rotation.

(c) Write down the order of rotational symmetry of
   (i) a rhombus (ii) a regular hexagon.

(d) The given shape could also represent a three dimensional shape. What is this shape?

Investigation

How many squares are there in the given figure?